Reseach Results

The branching coefficients of the tensor product of finite-dimensional irreducible $U_q(\mathfrak{g})$-modules, where $\mathfrak{g}$ is $\mathfrak{so}(2n+1,\mathbb{C})$ (B_n-type), $\mathfrak{sp}(2n,\mathbb{C})$ (C_n-type), and $\mathfrak{so}(2n,\mathbb{C})$ (D_n-type), are expressed as the sum of products of three Littlewood-Richardson (LR) coefficients in the stable region. I gave an interpretation of this formula on the branching rule by Kashiwara's crystal theory. Firstly, I constructed an explicit surjection from the LR crystal of type C_n to the disjoint union of Cartesian product of two LR crystals of A_{n-1}-type, where the cardinality of LR crystal of A_{n-1}-type and that of the kernel of the surjection yield the corresponding LR coefficients so that this surjection recovers the formula on the branching rule. This surjection was constructed by the involved combinatorial analysis of Kashiwara-Nakashima (KN) tableaux of C_n-type. Secondly, I proved that LR crystal of B_n-type and that of D_n-type are identical to the LR crystal of C_n-type in the stable region. These results of the second part were suggested by Professor Jae-Hoon Kwon of Seoul National University and were proven in the course of the discussions with him. The research work on this subject will be published in Math. J. Okayama University.

The second research subject was on the $\mathfrak{q}(n)$-crystals, i.e., crystals for the queer Lie superalgebra. I proved that the set of primed tableaux admits the $\mathfrak{q}(n)$-crystal structure by giving the explicit odd Kashiwara operators, which is the generalization of the result of Hawkes et al.[1] The forms of the highest and lowest weight vectors of primed tableaux were also given. By using these results, I proved that the set of signed unimodal factorizations with m factors of reduced words of the type B-Coxeter groups, which is denoted by $U_m^{B\pm}$, and that of flattened words of the type D-Coxeter groups, which is denoted by $U_m^{D\pm}$, admit the $\mathfrak{q}(m)$-crystal structure. I clarified that the connected parts of $U_m^{B\pm}$ (resp. $U_m^{D\pm}$) consist of signed unimodal factorizations of type B (resp. D) Coxeter-Knuth related reduced (flattened) words. We also give the explicit algorithms for odd Kashiwara operators on the element of $U_m^{B\pm}$, which are also applicable on the element of $U_m^{D\pm}$ without any alterations. The research work on this subject will be published in Publ. RIMS Kyoto University.

References

[1] G. Hawkes, K. Paramonov, and A. Schilling, Crystal analysis of the type C Stanley symmetric functions, *Electronic J. Combin.* **24** (2017) #P3.51.

[2] T. Hiroshima, Crystal interpretation of a formula on the branching rule of type B_n, C_n, and D_n, to appear in *Math. J. Okayama University.*

[3] T. Hiroshima, $\mathfrak{q}$-crystal structure on primed tableaux and on signed unimodal factorizations of reduced words of type B, to appear in *Publ. RIMS Kyoto University.*