(2) Study proposal Makiko Mase

(i) Objects of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces. We simply call an algebraic complex $K3$ surface a $K3$ surface.

It is known that geometry of $K3$ surfaces is related to singularity theory, symplectic geometry, and the homological mirror conjecture. In particular, invertible polynomials studied by Ebeling, Takahashi, and Ploog and many others are known to be associated to families of $K3$ surfaces.

We are interested in studying not only $K3$ surfaces indivisually, but also mirror symmetric phenomenon for families of $K3$ surfaces.

Problems

1. On duality of families of $K3$ surfaces and the coupling of singularities.
2. The homological mirror and strange duality of invertible polynomials.
3. Weierstrass semi-groups of pointed curves in a $K3$ surface.
4. On moduli space of maps between a $K3$ surface and a Lie group.

(ii) Study methods

<u>Problem 1</u> A coupling for singularities introduced by Ebeling can be projectivised as invertible polynomials in four variables that are weighted homogeneous in the list of Yonemura's 95. In this project, we consider whether or not the coupling can extend to the Batyrev-Borisov mirror symmetry, and the duality of Picard lattices for families of $K3$ surfaces thus obtained.

<u>Problem 2</u> The homological mirror conjecture is so interesting that connects the Fukaya category of a symplectic manifold and the derived category of an algebraic variety.

In this project, we consider the relations between the conjecture and the strange duality of bimodal singularities, the Batyrev-Borisov mirror, and the duality of Picard lattices for families of $K3$ surfaces.

We expect to give an systematical proof of the assertion that the strange duality for bimodal singularities extends to the duality of Picard lattices for some families of $K3$ surfaces, and a study of the Frobenius structure of the base space of "unfolding" for singularities.

<u>Problem 3</u> This is a joint work with Professor J.Komeda in Kanagawa Institute of Technology. For a given semigroup, it is important to find out a pointed curve that admit the semigroup as its Weierstrass semigroup.

We would like to characterise $K3$ surfaces by their subvarieties in terms of their Weierstrass semigroup by solving this problem for the $K3$ surfaces.

<u>Problem 4</u> Considering the definition, there naturally exists a map from an elliptic $K3$ surface to a Lie group. Being related to harmonic maps, it is important to study maps from a manifold to a Lie group in a prospect to differential geometry and differntial equations. We shall first characterise the moduli space of holomorphic maps from a $K3$ surface to a Grassmann variety, and then, we expand this to more general cases.

(iii) Aspects

We expect that we will be able to characterise many information of subvarieties of $K3$ surfaces, and study their configuration. We prospect the results of the study may be applied to that of algebraic cycles on projective models of $K3$ surfaces.