Research plan

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Now the questions on the smooth manifolds with trivial invariants are the main classical hard problems on 4-dimensional topology. Especially the following two conjectures are important: the smooth unknotting conjecture for surface knots saying that a smooth surface knot whose complement has abelian fundamental group is unknotted and the smooth 4-dimensional Poincare conjecture saying that a smooth 4-manifold homotopic to 4-sphere is diffeomorphic to 4-sphere.

More than 10 years I am studying how to solve the former problem. I use 2-dimensional braid theory cultivated by Professor Seiichi Kamada. In fact, when the complement of a given smooth 2-knot has an abelian fundamental group, there is a 1-parameter family of maps from the 2-sphere into the 4-space admitting only cusp birth and death as singularities connecting the given knot and the trivial knot. Moreover we transform it into a 1-parameter family of surface braids. To do so we need a generalized Markov type theorem which is applicable to the case admitting nodes. Professor Kamada is preparing a general paper with details.

Using the fact that each 2-dimensional braid is described by a 1-dimensional chart on 2-space, its 1-parameter family is described in the 3-space adding the time axis. Taking an appropriate cusp-birth we can make it to go down near the bottom which is a trivial 2-dimensional braid. Then there are no other double points in the levels between the cusp birth and death. Analyzing this situation by the braid group and local description of cusps, two ends of this part are isotopic after making connected sum with trivial torus for example. So, by induction the given knot becomes a trivial torus after making connected sum with one trivial torus. This is a result recently obtained assuming the Markov type theorem.

So, it is important to write a proof of the Markov type theorem and the above result. There would be multiple challenges. But it would be one of the best way to devote myself in the analysis of 1-parameter family of surface braid again.

To solve the big problem in this way I think I need various joint researches with Professor Kamada.