Research results achieved up to now

The following is my research results principally from 20 years ago to the present.

1) **Researches on Berezin transforms and on symmetry characterizations for homogeneous Siegel domains**. Berezin transforms are important operators appearing in Berezin quantization. However, they can be defined once we have reproducing kernel Hilbert spaces. In particular, I have been treating reproducing kernel Hilbert spaces of polynomials or holomorphic functions that carry representations of Lie groups. These Berezin transforms are selfadjoint operators commuting with group actions, and as such explicit descriptions of their spectral decompositions are significant problems. I did case studies in the joint papers with Etsuro Fujita (then a Master course student), and developed some general theory as well as describing explicit spectral decompositions in examples. After that, I proved that the Berezin transform on a homogeneous Siegel domain D commutes with the Laplace-Beltrami operator if and only if D is a symmetric domain. To show this, I generalize the Cayley transform of homogeneous Siegel domains introduced by Penney to parametrized ones and proved fundamental facts including that they are birational maps onto homogeneous bounded domains. Moreover, I established a norm equality which is a necessary and sufficient condition for a homogeneous Siegel domain to be symmetric. This features in a more geometric way than those obtained earlier by Dorfmeister and Satake in an algebraic and a structural manner. I also proved that the Poisson kernel on a homogeneous Siegel domain defined in terms of Szegö kernel à la Hua is annihilated by the Laplace–Beltrami operator, that is, it is a harmonic function, if and only if the domain is symmetric (in the paper a little bit stronger fact is proved by generalizing the Hermitian metric). These are very interesting results in that properties of operators determine the shape of the underlying domains. I also performed the research with Chifune Kai (then a Master course student) on characterizations of symmetric domains among homogeneous open convex cones. We established a characterization by using pseudoinverses which generalize the inverse of Jordan algebras, and a characterization of symmetric tube domains by the convexity of the Cayley transform images.

2) **Extraction of homogeneous open convex cones with good properties**. This research has been done by recognizing that the world of general homogeneous open convex cones is really a mixture of wheat and chaff, and needs an appropriate selection to develop a rich analysis on them. By joint works with Hideyuki Ishi we give examples of irreducible non-symmetric cones which are linearly isomorphic to dual cones. In particular, we give examples of arbitrary rank greater than or equal to 3.

3) **Studies on basic relative invariants associated with homogeneous open convex cones**. The basic relative invariants associated with homogeneous open convex cones are a system of irreducible polynomials which generalizes principal minors of matrices, and describes homogeneous open convex cones as their positive domain. By a joint work with Ishi in 2008, a system of inequalities is established for symmetric tube domains by using analytic continuations of the basic relative invariants. Moreover, we raised an example of non-symmetric homogeneous tube domain in which this system of inequalities holds, and mentioned that this example is a unique one among tube domains with dimensions less than or equal to 10. I next studied with Hideto Nakashima homogeneous convex cones obtained by selfadjoint representations of Euclidean Jordan algebras,

and wrote down explicitly the associated basic relative invariants. As a by-product, we got systematically a series of homogeneous open convex cones with good properties in relation to the theme 2) above. In more detail, it turned out that the dual cone of the homogeneous cone treated in the research has basic relative invariants with degrees $1, 2, \ldots, r$ (r is the rank of the cone). On the other hand, by a joint work with Takashi Yamasaki in 2016, it is proved that irreducible symmetric cones of rank r are characterized by the property that the degrees of basic relative invariants associated with the cones and their dual cones are both $1, 2, \ldots, r$.

4) **Studies on realizations of homogeneous convex cones**. In the joint work with Yamasaki in 2015, we realized general homogeneous open convex cones by using matrices. The idea generalized the original one of Vinberg who gave an example of non-symmetric homogeneous open convex cone for the first time. Our realization uses oriented graphs drawn by the algebraic data of given homogeneous open convex cones, and puts the general theory, which has been considered difficult so far, of homogeneous open convex cones in a black box. The realization is just a simple procedure, and makes the access to homogeneous convex cones easier to non-specialists.