

## Plan of Research

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### Litherland's Alexander polynomial for handlebody-knots

A genus  $g$  handlebody-knot is a genus  $g$  handlebody embedded in the 3-sphere. The Alexander polynomial is an invariant of a pair of handlebody-knot and its meridian system. Replacing a meridian system of a handlebody-knot acts on the Alexander invariant as an action of  $GL(g, \mathbb{Z})$ . We introduced an invariant  $G_H$  for handlebody-knots by using an invariant of the action of  $GL(g, \mathbb{Z})$  from the Alexander polynomial [2].

R. Litherland introduced the Alexander polynomial for  $\theta_g$ -curves [1]. In general, the elementary ideal of the Alexander invariant is not principal for  $\theta_g$ -curves. Thus, there are infinitely many  $\theta_g$ -curves whose Alexander invariant is non-trivial and Alexander polynomial is trivial. However, the elementary ideal of Litherland's Alexander invariant is principal, and Litherland's Alexander polynomial is non-trivial for  $\theta_g$ -curve.

We extended Litherland's Alexander polynomial of a  $\theta_g$ -curve to that a pair of  $H$  and its meridian system with base point and understood how act replacing a meridian system for Litherland's Alexander polynomial of handlebody-knot  $4_1$ . We would like to consider that how act replacing a meridian system for Litherland's Alexander polynomial of other handlebody-knots.

### Twisted Alexander polynomial for handlebody-knots

We have some property of irreducibility of  $H$  and constituent link of  $H$  by using the Alexander polynomial as previous research. I would like to expand this result for the twisted Alexander polynomial for a handlebody-knot.

## References

- [1] R. Litherland, The Alexander module of a knotted theta-curve, *Math. Proc. Camb. Phil. Soc.*, 106 (1989), 95–106.
- [2] S. Okazaki, An invariant coming from the Alexander polynomial for handlebody-knots, preprint.