The research proposal

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An approach to the theory of harmonic maps via exponentially harmonic maps

- In [5] of the papers list, the applicant proves the existence of a time-global solution to a time evolutional equation for exponentially harmonic maps into nonpositively curved manifolds, which is of the form:

$$\partial_t u = \Delta_g u + \langle \nabla |\nabla u|^2, \nabla u \rangle + A(u)(\nabla u, \nabla u). \tag{2}$$

  It is expected that the equation has a solution without the curvature assumption; indeed, he has obtained a gradient estimate for its solution in the most general case. One of the goals of this study is to find higher order estimate for its solution.

- An exponentially harmonic map flow, or a solution to (2), enables a refined analysis on the singular set of a sequence of exponentially harmonic maps approximating harmonic maps. The applicant will study the flow from a surface to detect bubbles which arise from the singular set, and study the structure of the moduli space of exponentially harmonic maps from a surface.

- A monotonicity formula for the flow (2) with respect to the radius of a geodesic sphere enables us to analyze the singularities of an approximating sequence of ε-exponentially harmonic maps, and to establish an existence theory of higher dimensional harmonic maps. To find the formula is one of the most important issues of this research.

- In [4], the applicant proves the constancy of certain exponentially harmonic maps with bounded exponential energy. The constancy of exponentially harmonic functions with bounded image is also expected and some partial affirmative answers (with additional technical conditions) are obtained. The applicant will try the technique in [4] to achieve a complete answer to this problem.

- In [7], a recipe of equivariant exponentially harmonic maps between spheres is obtained. Unlike the case of harmonic maps, it is proved that there always exists an equivariant exponentially harmonic map between spheres. The problem whether every homotopy class of maps between spheres is represented by a harmonic map remains to be proved. The applicant will give a new insight into the problem using equivariant exponentially harmonic maps.

A discrete surface theory for graphs

- In [3], the mean curvature and the Gauss curvature are calculated for the Goldberg-Coxeter (GC) constructions of several examples of discrete surfaces. GC constructions are considered as subdivisions of discrete surfaces and the applicant is interested in the convergence of them and their limit objects. The applicant will find a general theory for the convergence of GC constructions of discrete surfaces.

- In [6], eigenvalues of the Laplacian on the GC constructions for 3- and 4-valent finite graphs are studied, but the limit distribution of the eigenvalue distributions has not been obtained. To find the limit distribution is one of the goals of this research. It would not be so straightforward to apply a trace formula to obtain a limit distribution. So, the enumeration of closed walks in a GC construction would be the key to the success.