Plan of Study

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Elliptic Ruijsenaars system and a difference deformation of elliptic curves

The elliptic Calogero-Moser system is a quantum many-body system with elliptic potential. Let H^{CM} be the Hamiltonian of the system, and τ be the elliptic modulus. Then there exists a solution Ψ which satisfies

$$H^{\mathrm{CM}}\Psi=\frac{\partial\Psi}{\partial\tau}.$$

The emergence of τ-derivative is natural from the viewpoint of conformal field theory on elliptic curves.

On the other hand, the elliptic Ruijsenaars system is a q-deformation of the elliptic Calogero-Moser system. Except for special cases, it is difficult to construct solutions of the elliptic Ruijsenaars system. Recalling that the elliptic Calogero-Moser system has solutions with τ-derivative, we may expect that the elliptic Ruijsenaars system has solutions with shifts as $\tau\to\tau+\tau_0$. Since τ-derivative corresponds to infinitesimal deformation of elliptic curves, such shifts of τ can be regarded as a certain difference deformation of elliptic curves. The author intends to identify the difference defomation of elliptic modulus τ which should arise in the elliptic Ruijsenaars system.

The theory of q-KZB equation due to Felder-Varchenko can be a hint to study the elliptic Ruijsenaars system. Conformal field theory on elliptic curves is formulated by the theory of infinitesimal deformation of elliptic curves. Thus, the KZB equation, which are differential equations for the correlation function of conformal field theory on elliptic curves, include τ-derivative, and it is known that the equations have connections to the elliptic Calogero-Moser system. Felder-Varchenko studied a q-deformation of the KZB equations, and they also introduced q-KZB heat equation which includes shifts of τ. The author conjectures that a difference deformation of elliptic modulus τ resembling the theory of Felder-Varchenko arises in the elliptic Ruijsenaars system.