A summary of my research

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Outline of my research

Functional inequalities express an inclusion relation between functional spaces. Not only that it is one of fundamental tools to study differential equations. Therefore, it is important.

I study explicit value and minimization problems related to best constant of Hardy type inequalities and several functional inequalities. And I study existence of solution to elliptic equations. Recently, I am interested in the limiting case of the classical Hardy inequality, that is called the critical Hardy inequality：

$$\left(\frac{N-1}{N}\right)^N \int_\Omega \frac{|v(y)|^N}{|y|^N\left(\log\frac{R}{|y|}\right)^N}\,dy \le \int_\Omega |\nabla v(y)|^N dy \quad \left(\forall v \in W_0^{1,N}(\Omega), R := \sup_{y\in\Omega}|y|\right)$$

Details

The critical Hardy inequality is quite different from the subcritical (classical) Hardy inequality in the form, the best constant, and potential function. However, in the view of the attainability of each minimization problem, they are similar. Indeed, we showed an unexpected equivalence between the subcritical and the critical Hardy inequalities via a transformation (ref. [7]). In addition, I extended it Sobolev type inequalities and gave an answer to an open problem mentioned by Horiuchi-Kumlin's paper (2012) and Horiuchi's Japanese article (2016) (ref. [2]). Moreover, we studied the other Hardy type inequalities (Rellich inequalities, Caffarelli-Kohn-Nirenberg inequalities and so on) and their improvements (ref. [1], [5], [6], [9]) and their applications (ref. [3], [8]). Furthermore, we studied the compactness of the embedding from the radial Soboelv spaces $W^{1,p}_{\mathrm{rad}}(\mathbb{R}^N)$ to the variable Lebesgue spaces $L^{q(\cdot)}(\mathbb{R}^N)$. This problem is related to the non-compactness of vanishing phenomenon. Concretely, we showed that the compactness of the embedding depends on the speed of the variable exponent $q(\cdot)$ which goes to p at infinity. And, we applied it to an existence of a solution to an elliptic equation which does not satisfy Ambrosetti-Rabinowitz condition by variational method (ref. [4]).