# (2) Research plans

As a postdoctoral fellow, I will firmly continue to develop the following research lines:

- **Branching algorithm that tells us how many Killing vector fields exist**

  In the previous research, we found that *local curvature obstructions* to the existence of Killing vector fields can be identified by analysing the Killing equation. These obstructions give us the branching algorithm that tells us how many Killing vector fields exist for given metrics. We also found that such algorithms depend on the dimensionality of the manifold as well as its signature.

  I will consider the higher-dimensional (pseudo-)Riemannian manifolds in the future. A primary target is to construct the algorithms for Killing vector fields in 4 and 5-dimensional spaces/spacetimes. The reason is that General Relativity is usually studied on such manifolds and therefore it expects a huge demand for our algorithm.

- **Curvature obstruction to the existence of hidden symmetries**

  In the previous research, we have formulated the integrability conditions of the higher-order Killing tensor fields. So it is natural to ask ourselves whether we can identify local curvature obstructions for the Killing tensor fields. In the future research, I will also try to formulate local curvature obstructions for Killing tensor fields and then to construct the branching algorithms for them.

- **Integrability conditions for other types of overdetermined PDEs**

  We found that our prolongation procedure that exploits the Young symmetrisers can be extended to other types of overdetermined PDEs, e.g. the Killing-Yano equation, the $(p, q)$-type Killing spinor equation and massless higher spin field equations in 4-dimensional spaces, etc. On the other hand, I shall stress that the explicit forms of the integrability condition for exterior differential systems have brought us essential insights into General Relativity. The Raychaudhuri equation for geodesics and the Gauss–Codazzi equations for embedded hypersurfaces are typical examples. The explicit forms of both the equations allow us to derive the so-called singularity theorems and to study Hamiltonian formulation of General Relativity. Therefore, I will try to derive the explicit forms of the integrability conditions of other types of overdetermined PDEs and expect that these conditions bring us new insights into General Relativity.

- **Generalisation of the prolongation procedure via the Young symmetrisers**

  In order to analyse the conformal Killing equation, we will need a technique to decompose a tensor product into the irreducible representations of SO(n), which corresponds to the operation to take the trace of tensor fields. So far, we do not know such a technique. Similar issues occur in the case of conformal Killing-Yano tensors. So it will be necessary to incorporate the trace operation into our analysis. Such modifications will be considered in the future research.