Summary of my research

In my master thesis, I studied a decomposition theory of exponential operators and proposed [1] [1] a general scheme to construct independent determining equations for relevant decomposition parameters by using Lyndon words on a free Lie algebra.

In my Ph.D. thesis, I studied an analytic Bethe ansatz (ABA) and T-systems (a system of functional relations among transfer matrices of solvable lattice models in statistical mechanics). We gave solutions of the T-system for $U_q(D_r^{(1)})$ [2] and discretized affine Toda field equations [3]. I proposed [4-8] a general theory on ABA based on the Bethe ansatz equations associated with superalgebras. Based on the T-system proposed in [8], we derived [9-11,15] TBA equations (nonlinear integral equations for the free energy of quantum integrable systems at finite temperature).

We studied fermionic formulae and Q-systems related to completeness of the Bethe ansatz [12-14]. I also constructed [16] a Casorati determinant solution of a T-system.

In general, TBA equations is an infinite number of coupled nonlinear integral equations (NLIE) with an infinite number of unknown functions. Then I systematically derived NLIE with a finite number (= rank) of unknown functions, which are equivalent to the TBA equations, associated with Lie algebras of arbitrary rank [17,18,20,21,23]. They are applicable to calculations on thermodynamics of spin ladder systems. We had an excellent agreement with experimental results [19,25].

Evaluation of correlation functions in the statistical mechanics is a very difficult problem even for solvable lattice models. We succeeded to evaluate correlation functions at finite temperatures based on the high temperature expansion technique [22,24].

We constructed [26] Baxter Q-operators for $U_q(\hat{sl}(2|1))$ based on infinite dimensional representations of q-oscillator algebras. Our formulation can be applicable to both lattice models and the CFT.

I proposed [27] Wronskian-type solutions of the T-system for $U_q(\hat{gl}(M|N))$. I identified that the number of the Baxter Q-operators is 2^{M+N} for the first time. Based on techniques and ideas proposed in [27], we gave solutions of the T-, Q- and Y-system for the AdS/CFT [28,29,31].

We proposed [30] a new definition of Baxter Q-operators based on the 'co-derivative' on supercharacters of $gl(M|N)$, and derived Bethe ansatz equations without using the Bethe ansatz.

We proposed [32,33] the 'master T-operator', which is a sort of generating function for transfer matrices for any generalized quantum integrable spin chain. It corresponds to a τ-function of classical integrable hierarchies of soliton equations. Baxter Q-operators and functional relations in Hirota bi-linear form are systematically derived from it.

Based on properties of the universal R-matrix, we proved [34] the fact that L-operators for Verma modules for $U_q(\hat{sl}(2))$ factorize with respect to L-operators for Baxter Q-operators.

I studied asymptotic representations for the quantum affine superalgebra $U_q(\hat{gl}(M|N))$ and obtained solutions (L-operators) of the graded-Yang-Baxter equation related to Baxter Q-operators [35].

We proposed [37] a decomposition formula of Beisert's S-matrix for the AdS/CFT with respect to the R-matrix of the free fermion model and pointed out a relation to $U_q(\hat{sl}(2))$ at $q = i$.

In [36], we extended results in [32] to $gl(N|M)$ case. The zeros of the eigenvalues of the master T-operator obey an equation of motion for the Ruijsenaars-Schneider model. We also proposed a set of algebraic equations, which gives the eigenvalues of Hamiltonians for supersymmetric spin chains.

We solved [39] intertwining relations of the augmented q-Onsager algebra, and obtained generic boundary K-operators in terms of the Cartan element of $U_q(sl_2)$. Considering asymptotic representations of the algebra in Verma modules, we derived K-operators for Baxter Q-operators for integrable models with open boundary conditions. I generalized [40] K-operators in [39] to the higher rank case ($U_q(\hat{gl}(N))$). As there was no paper on a higher rank analogue of the augmented q-Onsager algebra, I derived intertwining relations from the reflection equation, and proposed commutation relations of the underlying symmetry algebra for symmetric tensor representations (and their infinite dimensional analogues).

I gave [38] a solution of the set-theoretic (quantum) Yang-Baxter equation as a product of quasi-Plücker coordinates (quasi-determinant) over a matrix composed of L-operators (which are images of the universal R-matrix for $U_q(gl(n))$). In the quasi-classical limit, this reduces to a new determinant (Plücker coordinate) expression of a classical Yang-Baxter map.

---

[1] See my publication list for the reference number of each paper.