

Research Plan

(1) (Equivariant Morse theory for GKM-manifolds)

I continue to study the existence of invariant Morse functions on GKM-manifolds.

Let a representation V be equivariantly embedded to an another representation W (the embedding is not necessary to be linear). Then, I consider whether one can construct a family of invariant functions over W such that each restriction $\Phi_\lambda|_V$ satisfies $\text{Cr}(\Phi_\lambda|_V) = \{0_V\}$.

If such a family of invariant functions exists, it is seemingly possible to construct invariant Morse functions on GKM-manifolds by using the family and the result stated in the research plan.

(2) (Freeness of graph equivariant cohomology)

Franz-Puppe proved that the localization theorem of Goresky-Kottwitz-MacPherson for integral equivariant cohomology hold for GKM-manifolds if each stabilizer group is connected and the equivariant cohomology $H_T^*(X, \mathbb{Z})$ is free over the polynomial ring $H_T^*(BT, \mathbb{Z})$.

As a combinatorial counterpart of this fact, we consider whether the graph equivariant cohomology $H_T^*(\mathcal{G})$ of a GKM-graph \mathcal{G} is a free module over the polynomial ring. Whereas Guillemin-Zara already mentioned this problem when they introduced the notion of a graph equivariant cohomology, any satisfactory answer is still not known at this time (it is known that $H_T^*(\mathcal{G})$ is a free module over the polynomial ring under a certain Morse theoretic assumption). I will consider this problem in view of the system of parameters.

(3) (Constructing non-singular complete GKM-varieties)

This research plan is aimed to establish a classification theory in GKM-theory. Although the class of GKM-varieties is a vast generalization of that of toric varieties, any classification result is still not known for GKM-varieties. This research plain is also motivated by cohomological rigidity problem for GKM-varieties.