Research result

(1) (**Invariant function on GKM-representation space**)

To establish existence theorem for invariant Morse functions on GKM-manifolds, I investigated the structure of an invariant function on a GKM-representation space. To prove the existence of invariant Morse function, it is necessary to clarify the structure of the Hessian at each fixed point. I proved that the Hessian is of diagonal type with respect to some suitable coordinate system on the representation space. This result can be thought as a GKM-theoretic analogue of the Morse lemma.

Surprisingly, in the course of proof of this fact the GKM-condition naturally arise. In particular, it turns out that the GKM-condition is of Morse theoretic nature.

(2) (**Graph equivariant cohomological rigidity for GKM-graphs**)

The equivariant cohomology of an equivariantly formal GKM-space is strongly connected to combinatorics (GKM-theory). One face of GKM-manifolds is that GKM-manifolds are generalization of toric manifolds, however, any classification theory in GKM-theory is still not established.

Accordingly, to build up classification theory in GKM-theory I proved a type of rigidity of GKM-graphs with respect to graph equivariant cohomology algebras (this gives a GKM-theoretic generalization of a result of Masuda).

For its proof it is important to introduce a new object called a 1-ideal. The notion of a 1-ideal is so simple, however, it provides an algebrization of the notion of the 1-skeleton of a GKM-manifold. Moreover it well-behaves under any algebra automorphism of graph equivariant cohomology algebra.

As a joint work with Matthias Franz, we found an another proof of the equivariant cohomological rigidity of GKM-graphs.