

# Research Results

Reiji Yoshioka

## • 2d/4d(5d) connection

The 2d/4d connection states the equivalence between the conformal block in 2d CFT and the instanton partition function in 4d supersymmetric gauge theory. The connection between the  $q$ - $W_n$  conformal block and the 5d instanton partition function was also proposed. These have two parameters  $q$  and  $t$ . By taking the limit  $q, t \rightarrow 1$ , this connection reduces to the 2d-4d one. I have studied the root of unity limit in  $q$  and  $t$ . The generators of the  $q$ -Virasoro ( $q$ - $W_2$ ) algebra can be described by a  $q$ -boson field. In the  $q, t \rightarrow -1$  limit, the generators of the superconformal algebra appear. Similarly, the free boson and free fermion which describe this algebra can be obtained from the  $q$ -boson in this limit. On the 5d side, I obtained the 5d instanton partition function in the root of unity limit which is the same as that used on the 2d side. I have confirmed that the results are equal to the 4d ALE instanton partition function at the lower level at least. The 2d-4d connection can be understood through the limiting procedure in the 2d/5d connection.

In the general  $r$ -th root of unity limit of  $q$ - $W_n$  algebra, the  $\mathbf{Z}_r$ -parafermions appear. The obtained theory is the coset CFT which has  $\frac{\widehat{sl}(n)_r \oplus \widehat{sl}(n)_p}{\widehat{sl}(n)_{r+p}}$  symmetry. In fact, the central charge of the energy-momentum tensor is exactly reproduced. The parameter  $p$  is related with the omega-background of the corresponding gauge theory and its relation was also clarified.

On the other hand, the  $q$ -deformation has a kinds of ambiguity. This is obstacle for the clear expression of the vertex operator of  $q$ -Virasoro algebra. Based on the 2d/5d correspondence, the  $q$ -deformed vertex operator was obtained.

The screening currents play the important role to construct the integral representation of the conformal block. These currents are also the generators of the elliptic algebra  $U_{q,p}(\mathfrak{sl}(2))$ . I studied the features of the root of unity limit of the elliptic algebra.

I considered also another root of unity limit ( $q \rightarrow 1, t \rightarrow -1$ ). There exists the correspondence between a modified affine  $sl(2)_k$  current block and the instanton partition function in the presence of a surface operator. The representation of affine  $sl(2)_k$  algebra can be realized in terms of free fields which are obtained from the  $q$ -boson at the root of unity limit as mentioned above. I presented explicitly the free field representation of affine  $sl(2)_k$  algebra and derived the integral representation of the modified current blocks.

## • Matrix Model

Since the superstring theory is defined on ten-dimensional spacetime, so are the matrix models. In conclusion, if we would like to obtain the models which describe realistic world, the compactification of spacetime to four-dimensions are required. I studied about the USp matrix model compactified by  $\mathbb{C}^3/\mathbb{Z}_3$  and then I succeeded in enumerating all possibilities to be able to be consistently defined.

In the matrix model, the spacetime points are described by the eigenvalues of the bosonic matrices. Therefore, the matrix model has possibility to describe the spacetime where we live. By calculating the one-loop effective action for the eigenvalues I showed that the spacetime directional asymmetry of attractive force between eigenvalues appears. The spacetime points are eventually attractive to an imaginary four-dimensional surface. Moreover I calculated two-loop corrections and found that in the case of short distance the interaction between eigenvalues is turned to repulsive. In the USp matrix model, the spacetime points are stabilized near above mentioned four-dimensional plane and then the 4-d spacetime are produced.