Research Plan

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(I) A study of Iwanaga-Gorenstein algebras of finite Cohen-Macaulay type.

An algebra having self-injective dimension on both sides $d < \infty$ is called Iwanaga-Gorenstein (=IG). The concept of IG algebras is a generalization of that of commutative Gorenstein rings and of self-injective algebras in common. And some experts have studied these IG algebras. However, there are many unknown things in representation theory of IG algebras. The important property of an IG algebra is the following: the category of Cohen-Macaulay(=CM) modules (which are special modules) forms a Frobenius category, and then its stable category forms a triangulated category. An IG algebra is called *of finite CM type* if there are only finitely many isomorphism classes of indecomposable CM modules. The main subject of this project is a construction and a classification of IG algebras of finite CM type. The class of these algebras is the most basic in the class of all IG algebras. This project must be regarded as a generalization of the theory of self-injective algebras due to Tachikawa and Riedtmann. In the sequel, we assume that the self-injective dimensions are at most $d = 1$.

(I)-1 Construction of IG algebras of finite CM type:

Let A be a hereditary algebra of Dynkin type, C an A-A-bimodule and $R(A, C)$ the repetitive algebra of A by C. First, we will investigate the question whether the class of the orbit algebras $R(A, C)/G$ of $R(A, C)$ for some group G should give a certain class of IG algebras of finite CM type. In the case $C = D(A)$ where $D = \mathrm{Hom}_k(\text{-}, k)$ with the base field k and G is the cyclic group generated by the Nakayama automorphism of $R(A, C)$, the algebra $A \ltimes C = R(A, C)/G$ is just the trivial extension algebra of A by C, and hence it is a self-injective algebra. Here, note that $D(A)$ is an injective cogenerator. The concept of cotilting modules is naturally considered as a generalization of that of injective cogenerators. Therefore, we will consider the case that C is a cotilting module such that the endomorphism algebra of C is also isomorphic to A.

(I)-2 Classification of AR-quivers of the category of CM modules over IG algebras of finite CM type:

We consider a certain assumption automatically holds in the case of self-injective algebras. Under this assumption, the AR-quiver of the category of CM modules over an IG algebra of finite CM type is given by the translation quiver $(\mathbb{Z}\Delta)_{\mathcal{C}}/G$ obtained from some Dynkin diagram Δ, a set $\mathcal{C}$ (*configuration*) of vertices in $\mathbb{Z}\Delta$ and the automorphism group $H(H\Delta = \Delta)$ of $\mathbb{Z}\Delta$ similarly to the case of self-injective algebras. Then, the problem is to characterize the configuration $\mathcal{C}$ in combinatorics. Second, we will consider this problem, considering the case of self-injective algebras.

(I)-3 Classification of IG algebras of finite CM type:

We mainly consider the *standard* case. An IG algebra is called *standard* if the category of indecomposable CM modules is equivalent to the mesh category of the AR-quiver of the category of CM modules. In this case, we will actually calculate algebras from AR-quivers which were classified in **(I)-2**. Thus, the third purpose is to investigate whether all standard IG algebras of finite CM type should be given in the way of **(I)-1**. If there is an IG algebra of finite CM type which is not given in the way of **(I)-1**, then we will consider the more general construction containing such an algebra.

(II) A study of applications to Topological data analysis(=TDA).

I am a member of TDA Team (Team Leader: Y. Hiraoka) belongs to Center for Advanced Intelligence Project, RIKEN. In recent years, the use of TDA to understand the shape of data has become popular, with persistent homology. Persistent homology is used to study the persistence of topological features such as holes in a one-parameter increasing family of spaces. These features are summarized in a persistence diagram, a compact descriptor of the topological features. This is enabled by the result that any 1D persistence module(=persistent homology) can be decomposed into intervals. The focus on one-parameter families is a limitation of the current theory. While there is a need for practical tools applying the ideas of persistence to multiparametric data, multidimensional persistence modules are known to be difficult to apply practically. Thus, I will study the subject of the representation theory on multidimensional persistence modules.

(II)-1: Fourth, I would like to make a further investigation of our result [10] on multidimensional persistence modules.

(II)-2: Fifth, I will study an algebraic generalization of 'Stability theorem', which is one of the foundations of TDA with 1D persistence module. This endpoint is to extend this result to the multidimensional version.