Sukuse Abe

[Further research plan] The objective of this research is **that we make coloring invariants and quantum $U_q(\mathfrak{sl}_2)$ invariants stronger , categorize "Objects" and apply them to Low dimentional manifolds.** Here, the "objects" indicate various things such as virtual knots, surface knots, surface links and handlebody-knots. We aim to define quantum invariants for those various objects, and to unify and classify the existing invariants. The invariants and those correlations in the following figure 1 will guide our study. Normal knot theory has correlations similar to this figure 1. We aim at the development similar to the normal knot theory for various " objects. "

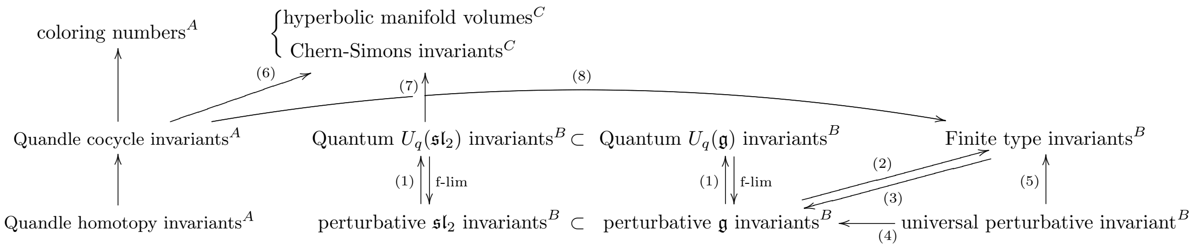

Figure 1: Relation between invariants

A superscript A represents coloring invariants with quandle. Other superscripts B and C represent quantum invariants and hyperbolic geometry invariants respectively. Moreover, an arrow in the figure shows that invariants at the head of the arrow can be gained from those at the tail: invariants at the tail of the arrow are stronger than those at the head. Furthermore, $\subset$ represents the relation of inclusion between two invariants. (1)–(7) of the figure 1 show the applicant's conjectures. The arrow (8) was demonstrated by the applicant (we obtained Vassiliev invariants form Quandle (shadow) cocycle invariants in the previous chapter (3)).

The applicant succeeded in defining quantum $U_q(\mathfrak{sl}_2)$ invariants and quantum $U_q(\mathfrak{so}_N)$ for virtual knots (see the report in "Musubime no suuri II"). We also succeeded in defining quantum $U_q(\mathfrak{g})$ invariants of virtual knots for any semi-simple Lie rings $\mathfrak{g}$ when the equation in the following figure 2 holds. Therefore, first of all, we will consider how to define the quantum $U_q(\mathfrak{g})$ invariants

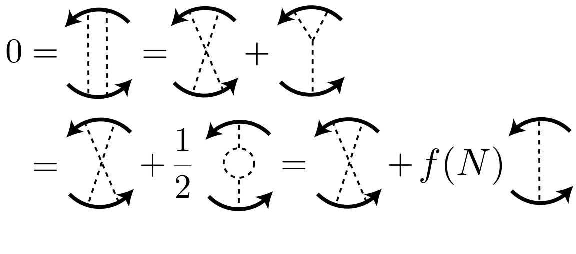

Figure 2: Weight system $W_{\mathfrak{g},R} : \mathcal{A}(S^1) \to \mathbb{C}[N, \hbar]$

for any semi-simple Lie rings $\mathfrak{g}$ when this equation does not hold.