Sukuse Abe

[Our previous research]

[(1) On 4-cocycles of Alexander quandles on finite fields] Quandle shadow cocycle invariants are invariants of oriented surface-links. To calculate these invariants, we need concrete quandle 4-cocycles. For Alexander quandle X on the finite field, we represented a nontrivial 4-cocycle as a polynomial, and when this quandle is $H^2_Q(X;\mathbb{Z}) \cong 0$, we decided $H^4_Q(X;\mathbb{Z})$. This research has been developed by Nosaka Takefumi. In case of Alexander quandles on the finite field (generally regular quandles can be used.), by the universal coefficient theorem and the hurewicz fundamental homomorphism theorem, we gained $H^2_Q(X;\mathbb{Z}) \cong 0 \Rightarrow H^4_Q(X;\mathbb{Z}) \cong \pi_3(BX)$, and thus the generating elements of quandle homotopy invariants of surface links were decided.

[(2) Willerton conjecture] Generally, it is known that when for the normalized prime vassiliev invariant v_d of the degree d, the knot K has a diagram of n crossings, the value of $v_d(K)$ is shown by the order of n^d. Therefore, the set $\left\{ \left(\frac{v_2(K)}{n^2}, \frac{v_3(K)}{n^3} \right) \in \mathbb{R}^2 \middle| K \text{ has a knot diagram with } n \text{ crossings} \right\}$ is bounded. For some knots, points are plotted, and then a fish-link graph appears. We found what shape this graph could be for torus knots. Moreover, for these knots, we completely solved the problems Willerton conjecture posed.

[(3) Relationship between quandle (shadow) cocycle invariants and finite type invariants] The relation between quandle (shadow) cocycle invariants and quantum invariants is not well known except the set the relation of the set-theoretic Yang-Baxter equation. We substitute $t = e^{a\hbar}$ and $\omega = e^{b\hbar}$ in

$$\Phi_f(L) \in \mathbb{Z}[\mathbb{F}_q] \cong \mathbb{Z}[t]/(t^p - 1, g(t), h_1(\omega), \ldots, h_\ell(\omega)).$$

Here, $g(t)$ is the polynomial on the finite field $\mathbb{F}_p$ and $\mathbb{F}_q \cong \mathbb{F}_p[t]/(g(t))$ and $h_1(\omega), \ldots, h_\ell(\omega)$ are relations of ω by coloring L. Furthermore, there exist positive integer a, b such that $g(e^{a\hbar}) = h_1(e^{b\hbar}) = \cdots = h_\ell(e^{b\hbar}) = 0$. We obtain following power series

$$\Phi_f(L)|_{t=e^{a\hbar},\omega=e^{b\hbar}} \mapsto \sum_{d=0}^{\infty} d!u_d(L)\hbar^d \in \mathbb{Z}/p\mathbb{Z}[[\hbar]].$$

Then, $(d!u_d(L) \bmod p) \in \mathbb{Z}/p\mathbb{Z}$ is a Vassiliev invariant of degree d of L. It was important that we consider the target set of Vassiliev invariants to be $\mathbb{Z}/p\mathbb{Z}$.

[(4) New quantum $U_q(\mathfrak{g})$ invariants of genus 2 handlebody-2-tangles] We gained the new quantum $U_q(\mathfrak{g})$ invariants and of genus 2 handlebody-2-tangles. These new invariants have the following characteristics. Although the two genus 2 handlebody-2-tangles to which a complement is homeomorphic cannot be distinguished by using usual quantum $U_q(\mathfrak{sl}_2)$ invariants. Those genus 2 handlebody-2-tangles can be distinguished by using newly defined quantum $U_q(\mathfrak{sl}_2)$ invariants. These invariants enable us to define the pertubative $\mathfrak{g}$ invariants and universal perturbative invariants and make it easy to calculate the pertubative $\mathfrak{sl}_2$ invariants. They can be applied to Representation theory and Prion graphs.