Research plan

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I will construct the theory of Abelian functions for higher genus curves based on the multivariate sigma functions.

1. Construction of the deformation of KdV-hierarchy.

V. Buchstaber, V. Enolski, and D. Leykin showed that the Abelian functions defined by the logarithmic derivatives of the hyperelliptic sigma functions satisfy the KdV-hierarchy by using the Abel-Jacobi map between the g-th symmetric products of a hyperelliptic curve of genus g and the Jacobian of the curve. I considered this method for the 2-symmetric products of a hyperelliptic curve of genus 3 and derived two parametric deformation of the KdV-equations, which are joint work with V. Buchstaber. I will generalize these results to the k-th symmetric products of a hyperelliptic curve of genus g, where $k < g$. Let $W(g, k)$ be the image of the Abel-Jacobi map of the k-th symmetric products of a hyperelliptic curve of genus g. I will show that the field of rational functions on the k-th symmetric products of the hyperelliptic curve of genus g is isomorphic to the field of meromorphic functions on $W(g, k)$. From these results, I will derive the new differential equations, which are deformation of KdV-hierarchy and integrable by the meromorphic functions on $W(g, k)$.

2. Inversion problem of the hyperelliptic integrals of any genus

I will consider the inversion problem of hyperelliptic integrals of genus g. I considered the solution of the inversion problem of the hyperelliptic integrals of genus 2 and its degeneration, which are joint work with V. Buchstaber. I will generalize these results to higher genus curves. First, I will describe the solution of the inversion problem of the hyperelliptic integrals of any genus in terms of the hyperelliptic sigma functions. Next, I will derive the differential equations satisfied by the solution of the inversion problem and recurrence relations of the coefficients of the series expansion of the solution. When we degenerate the curves, I will consider the degenerate limit of the solution of the inversion problem.

3. Hurwitz integrality for the sigma functions of the telescopic curves

The coefficients of the series expansion of the sigma functions for the telescopic curves are polynomials over the rationals of the coefficients of the defining equations of the curves. Y. Onishi showed that the series expansion of the sigma functions for the (n, s) curves is Hurwitz integral over the ring generated by the coefficients of the defining equation over the integers. From these results, we can treat the series expansion of the sigma functions not only over the complex numbers but also over p-adic numbers or other rings. Onishi proved these results by using the expression of the sigma functions for the (n, s) curves in terms of the tau functions of KP-hierarchy. This expression is given for the telescopic curves. By using this expression, I will show that the series expansion of the sigma functions for the telescopic curves is Hurwitz integral over the ring generated by the coefficients of the defining equations over the integers.