Previous research

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The elliptic sigma functions have the algebraic properties which directly related with the defining equation of the elliptic curves. F. Klein constructed the multivariate sigma functions associated with the hyperelliptic curves. For hyperelliptic curves of genera 2 and 3, H. Baker obtained explicit expressions for higher logarithmic derivatives of sigma functions of many variables in the form of polynomials in the second and third logarithmic derivatives of these functions. It was shown that these differential polynomials give fundamental equations of mathematical physics, including the KdV and KP equations. Recently, by the study of V. Buchstaber, V. Enolskii, and D. Leykin, the theory of the sigma functions was developed and many applications to integrable systems were obtained. The hyperelliptic sigma functions were generalized to the more general plane algebraic curves called (n, s) curves. I have studied the theory of the sigma functions and the Abelian functions and obtained the following results.

1. I generalized the sigma function to the telescopic curves which include the (n, s) curves (list of paper 1-5). I generalized the Jacobi inversion formulae to the telescopic curves (list of paper 1-4). I gave the expression of the tau functions of KP-hierarchy in terms of the sigma functions for the telescopic curves and derived the addition formulae for the sigma functions of the telescopic curves, which are joint work with A. Nakayashiki (list of paper 1-6).

2. The zero set of the sigma functions is called sigma divisor. I constructed the field of meromorphic functions on the sigma divisor of the hyperelliptic curves of genus 3. By the Abel-Jacobi map, the rational functions on the symmetric squares of the hyperelliptic curves of genus 3 correspond to the meromorphic functions on the sigma divisor of the hyperelliptic curves of genus 3. By using this correspondence, I constructed the solutions of the new dynamical systems introduced by Buchstaber and Mikhailov, which are joint work with V. M. Buchstaber (list of paper 1-3).

3. By using the above dynamical systems, I constructed the two parametric deformation of the KdV-hierarchy and its solution in terms of the meromorphic functions on the sigma divisor of the hyperelliptic curves of genus 3, which are joint work with V. M. Buchstaber (list of paper 1-2). From this result, we can find the relations between the dynamical systems of Buchstaber and Mikhailov and the fundamental equations of mathematical physics.

4. I considered the inversion problem of the hyperelliptic integrals of genus 2 and derived the recurrence relations on the coefficients of the series expansions of the solution of the inversion problem. When the curves of genus 2 deform to elliptic curves, I showed that the solution of the inversion problem transforms into the Weierstrass elliptic function, which are joint work with V. M. Buchstaber (list of paper 1-1). From this result, we can find a new relation between the degeneration of the curves and the degeneration of the sigma functions and the Abelian functions.