Future works

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As a generalization of the result in [2], it turns out that the best constant of higher dimensional Hardy-Leray inequality can also be computable for solenoidal fields without any symmetry assumption, and I am writing a paper for submission about this fact. For further development, there exist the following topics:

- My preceding research does not mention the remainder term that gives an estimate of the difference between the two sides of Hardy-Leray inequality. In future I will concern this topic, and I am planning to obtain an improved version of Hardy-Leray inequality that has a remainder term giving a stronger information about the attainability of the equality sign.

- An intermediate inequality between Hardy and Rellich inequalities, shown by Tertikas-Zographopoulos, seems to have an improvement by solenoidal or curl-free condition, and I am planning to compute the improved best value of the constant.

- My research so far have concerned only L^2 type of Hardy-Leray inequality, but the general L^p type for $p \neq 2$ also has a curl-free or solenoidal improvement. The computation of the best constant for $p \neq 2$ is not yet carried out and remains very difficult, since the method of spherical harmonics expansion is not applicable at all; I am seeking for the approach by constructing a rearrangement technique subject to not radial but axial symmetrization.

- My research so far have treated Hardy-Leray inequality on the whole Euclidean space, while the inequality with trace remainder term on the half space is known as Kato's inequality. I am interested in the problem whether this inequality for vector fields is more sharpened by assuming the curl-free or solenoidal condition.

- Uncertainty principle inequality together with its best constant is well known, which is famous in quantum mechanics. This inequality is similar to Hardy-Leray inequality. However, the solenoidal improvement is unknown, and it is of great interest to me.

Besides above, I am also looking into an application of Hardy-Leray inequality to Navier-Stokes equations; by way of the solenoidal improvement, it can be expected to give a new perspective around the Leray's existence theory for weak solutions.