Preceding results

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I study functional inequalities with focus on the so-called Hardy-Leray inequality for vector fields. This inequality, proved by G. H. Hardy for the one-dimensional case, was developed to the higher-dimensional case including the best constant, by J. Leray in the context of Navier-Stokes equations, and has been applied to many directions.

The problem of special interest for me is whether the best constant of the Hardy-Leray inequality exceeds the original one when we assume that the test vector fields are curl-free or solenoidal. This problem is a generalization of the one suggested by Costin-Maz'ya in 2007 for the condition of solenoidal, who derived the new best constant of Hardy-Leray inequality for solenoidal fields. However, in the process of this derivation, the condition of axisymmetry was additionally assumed in order to make computations easier, and it remained unsolved whether we can compute the best constant without such an additional condition. In this situation, I have ever obtained the following results:

Sharp Hardy-Leray inequality for three dimensional solenoidal fields

For the case of three-dimensional solenoidal fields, I tried, in a joint work with Prof. F. Takahashi [3], to weaken the axisymmetry condition which Costin-Maz'ya used to derive the sharp Hardy-Leray inequality. As a result, we found that the same best constant as Costin-Maz'ya can be obtained by restricting the axisymmetry condition on only the azimuthal components of the test fields. Furthermore, I concluded in [2], by giving an appropriate L^2 decomposition of solenoidal fields, that the same best constant can be obtained even if we do not assume the axisymmetry condition at all.

Sharp Rellich-Leray inequality for axisymmetric solenoidal fields

I derived in the paper [1] the best constant of Rellich-Leray inequality for axisymmetric solenoidal fields, which is a second-order version of Hardy-Leray inequality. Incidentally, I gave a characterization of axisymmetric vector fields to partially modify the Costin-Maz'ya's computation result for Hardy-Leray inequality with some weights, by showing that any higher dimensional axisymmetric field has no azimuthal component.

Sharp Hardy-Leray and Rellich-Leray inequalities for curl-free fields

If we restrict ourselves to the case of the two-dimensional case, the sharp Hardy-Leray inequality for solenoidal fields derived by Costin-Maz'ya is equivalent to that for curl-free fields. I extended this inequality to the higher-dimensional one for curl-free fields, as a joint work with Prof. F. Takahashi [4], and moreover derived the best constant of Rellich-Leray inequality for general dimensional curl-free fields.