Works

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A *link* is the union of mutually disjoint, finite number of oriented 1-sphere(s) in the 3-sphere S^3. In particular, one component link is called a *knot*. In general, knots and links are drawn in the plane (, more generally, for surfaces) as *link diagram*. More precisely, for a link in the 3-dimensional space or the 3-sphere, the image of projection of it contains multiple points. An image is said to be a *projection*, if the set of the multiple points consists of finitely many transverse double points. Each double point c of the image is called a *crossing*. Note that for a small disk neighborhood of a crossing, the crossing is formed by two arcs $a+$ and $a-$. In order to distinguish upper/under information of $a+$ and $a-$, we erase the arc which is lower than the other. We call this figure a *link diagram* of the link. Let D be a link diagram. Then $|D|$ denotes a projection of link and this is regarded as a 4-valent plane graph. Each component of $S^2\setminus|D|$ is called a *region* of D. It is one of the standard styles for studying the structure of knots and links, to apply local transformation on link diagram (we say that such local transformation an *operation*).

In 2010, Kengo Kishimoto introduced an operation called region crossing change. Let D be a link diagram. For a region R of D, $D_{rcc}(R)$ denotes the link diagram obtained from D by changing the upper/under information of the crossings on ∂R. We say that $D_{rcc}(R)$ is obtained from D by a region crossing change at R. In general, for a set H of regions of D, the link diagram obtained from D by *region crossing change* (denoted by r.c.c.) at H is defined to be the link diagram obtained by the composition of r.c.c's of each element. Then Ayaka Shimizu proved that r.c.c. is an unknotting operation in [S]. In fact, in [S], she showed that any set of crossings of each knot diagram can be changed by a r.c.c. Further in general, it is remarked that for a link diagram D, a link diagram of the trivial link cannot be obtained from D by any r.c.c. Recently Ayumu Inoue-Ryo Shimizu [IS] introduced another operation called region freeze crossing change related to region crossing change. Let D be a link diagram. For a region R of D, $D_{rfcc}(R)$ denotes the link diagram obtained from D by changing the upper/under information of the crossings not on ∂R. We say that $D_{rfcc}(R)$ is obtained from D by a region freeze crossing change at R. In general, for a set H of regions of D, the link diagram obtained from D by *region freeze crossing change* (denoted by r.f.c.c.) at H is defined to be the link diagram obtained by the composition of r.f.c.c.'s of each element. They showed that there exists a knot diagram such that some changes of crossings cannot be realized by r.f.c.c.

We induced a $\mathbf{Z}_2$-linear map Φ from the set based on the regions to the set based on the crossings defined by r.c.c. and made use it to study r.c.c. In fact, we have the following results.

- The trivial transformation of r.c.c. corresponds to $\ker\Phi$. Then we gave a neat representative of basis of $\ker\Phi$[H1].
- The image of Φ is a linear space. Then we gave a geometric generator of $\mathrm{Im}\Phi$. Further we studied the cokernel of Φ. Then we gave a method giving a representative of $\mathrm{Coker}\Phi$ by using a graph obtained from given link diagram [H2].

Next, we induced a $\mathbf{Z}_2$-linear map Ψ from the set based on the regions to the set based on the crossings defined by r.f.c.c. and made use of it to study r.f.c.c. The trivial transformation of r.c.c. corresponds to $\ker\Psi$.

- We proved that $\dim(\ker\Phi) \leqq \dim(\ker\Psi)$, and a necessary and sufficient condition for "<".
- We showed a necessary and sufficient condition that a given set of crossings is in $\mathrm{Im}\Phi\setminus\mathrm{Im}\Psi$.

These give a generalization of a result of Inoue-Shimizu's for link diagram.

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