Research Plan

Let $\mathfrak{g}$ be an affine Lie algebra. For $\mathfrak{g}$ of nonexceptional types, the Kerov-Kirillov-Reshetikhin (KKR) type bijection Φ from rigged configurations to tensor products of Kirillov-Reshtikhin (KR) crystals associated with $\mathfrak{g}$ is established and a uniform description of the bijection Φ is also given [1, 2].

On the other hand, there are several unsolved problems on the KKR type bijection Φ for $\mathfrak{g}$ of exceptional types. In particular, the explicit form of KKR type bijection of adjoint KR crystals is not known for $\mathfrak{g} = G_2^{(1)}$, which is one of the affine Lie algebras of untwisted non-simply-laced types. T. Scrimshaw posed the following problem [2].

Problem. Describe explicitly the map δ_θ for $\mathfrak{g}$ of untwisted non-simply-laced affine type.

Here, δ_θ is a building block of the bijection Φ. This problem was posed two years ago but is still open. Recently, the author have given heuristically a conjecture on the explicit form of Φ for $\mathfrak{g} = G_2^{(1)}$. The conjectural form of Φ is a bunch of complicated combinatorial rules and is completely different from the KKR type bijections of other types discovered so far. The research plan in 2020 is to prove the conjecture and to establish the KKR type bijection for $\mathfrak{g} = G_2^{(1)}$.

References

[1] M. Okado, A. Schilling, and T. Scrimshaw, Rigged configuration bijecttion and proof of the $X = M$ conjecture for nonexceptional affine types, J. Algebra **516** (2018) 1–37.

[2] T. Scrimshaw, Uniform description of the rigged configuration bijection, arXiv:math/1703.08945v1.