

Research Results

The branching coefficients of the tensor product of finite-dimensional irreducible $U_q(\mathfrak{g})$ -modules, where \mathfrak{g} is $\mathfrak{so}(2n+1, \mathbb{C})$ (B_n -type), $\mathfrak{sp}(2n, \mathbb{C})$ (C_n -type), and $\mathfrak{so}(2n, \mathbb{C})$ (D_n -type), are expressed as the sum of products of three Littlewood-Richardson (LR) coefficients in the stable region. In [3] an interpretation of this formula on the branching rule in terms of Kashiwara crystals.

The second research subject concerns $\mathfrak{q}(n)$ -crystals, i.e., crystals for the queer Lie superalgebra. In [1] the author proved that the set of primed tableaux admits the $\mathfrak{q}(n)$ -crystal structure by giving the explicit odd Kashiwara operators. The forms of the highest and lowest weight vectors of primed tableaux were also given. By using these results, the author proved that the set of signed unimodal factorizations with m factors of reduced words of the type B -Coxeter groups, which is denoted by $U_m^{B\pm}$, admits the $\mathfrak{q}(m)$ -crystal structure. It was clarified that the connected parts of $U_m^{B\pm}$ consist of signed unimodal factorizations of type B Coxeter-Knuth related reduced words.

The third research subject also concerns $\mathfrak{q}(n)$ -crystals, which are called queer supercrystals in [2]. It was shown that the set of increasing factorizations of fixed-point-free (FPF) involution words has the structure of queer supercrystals [2]. By exploiting the algorithm of symplectic shifted Hecke insertion recently introduced by Marberg [4], the author established the one-to-one correspondence between the set of increasing factorizations of FPF involution words and the set of primed tableaux (semistandard marked shifted tableaux) and the latter admits the structure of queer supercrystals. In order to establish the correspondence, the author proved that two FPF-involution words are Coxeter-Knuth related if and only if they yield the same insertion tableau in the symplectic shifted Hecke insertion, where the insertion tableau is an increasing shifted tableau and the recording tableau is a primed tableau.

References

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- [2] T. Hiroshima, Queer supercrystal structure for increasing factorizations of fixed-point-free involution words, arXiv:math/1907.10836v1.
- [3] T. Hiroshima, Crystal interpretation of a formula on the branching rule of type B_n , C_n , and D_n , Math. J. Okayama Univ. **62** (2020) 87–178.
- [4] E. Marberg, A symplectic refinement of shifted Hecke insertion, arXiv:math/1901.06771v3.