Research Plan

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1. The structure of weakly reflective submanifolds

Weakly reflective submanifolds, introduced by Ikawa, Sakai and Tasaki, are minimal submanifolds in Riemannian manifolds which have certain symmetries and are known as interesting objects. Several kinds of examples of them are given as orbits of isometric actions. However many properties of them have not been well-studied yet. In my research it was shown that from weakly reflective submanifolds in compact normal homogeneous spaces, weakly reflective PF submanifolds in Hilbert spaces are obtained (result (b)). Although so obtained weakly reflective PF submanifolds are infinite dimensional they are expected to have simple structures due to linearity of Hilbert spaces. The purpose of this research is to study the structure of finite dimensional weakly reflective submanifolds via this infinite dimensional linearlization.

2. Minimal orbits in infinite dimensional hyper polar actions

It is an interesting problem for a given isometric action to investigate its minimal orbits. In particular *hyperpolar actions* are isometric actions highly related to symmetric spaces and are well-studied in finite dimensional cases. However in infinite dimensions their minimal orbits are not so studied yet. The purpose of this research is to classify minimal orbits in $P(G,H)$*-actions*, which are infinite dimensional hyperpolar actions on Hilbert spaces. Several examples of minimal PF submanifolds have been obtained as orbits of hyperpolar $P(G,H)$-actions (result (b)). From this research more examples of minimal PF submanifolds are expected to be obtained.

3. Isotropy representations of affine Kac-Moody symmetric spaces

Affine Kac-Moody symmetric spaces are infinite dimensional symmetric spaces proposed by Terng and established by Heintze, Popescu and Freyn based on Kac-Moody theory. Many similar properties between those spaces and finite dimensional Riemannian symmetric spaces are known. In particular their isotropy representations are given by hyperpolar $P(G,H)$-actions mentioned above. The purpose of this research is to study minimal orbits of isotropy representations of affine Kac-Moody symmetric spaces and to clarify similarities to Riemannian symmetric spaces.

4. Reformulation of integrable systems by affine Kac-Moody groups

According to the study by Kashiwara, Jimbo, Date and Miwa symmetry of soliton equations are described in terms of affine Kac-Moody algebras. On the other hand Terng and Uhlenbeck showed that transformations on solution spaces of some integrable equations can be described by loop group actions. An *affine Kac-Moody group* is a Frechet Lie group corresponding to an affine Kac-Moody algebra and is realized as a T^2-bundle over a twisted loop group of smooth loops. In the study of affine Kac Moody symmetric spaces mentioned above it has become clear that an affine Kac-Moody group has a tame Frechet manifold structure. We aim to integrate the above two methods for integrable systems by affine Kac-Moody groups.