Research Achievements

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1. Overview

One generalization of submanifolds in Euclidean spaces is given by submanifolds in finite dimensional Riemannian manifolds. Another generalization is given by submanifolds in Hilbert spaces which can be of infinite dimension. In my research I introduced the concepts of minimal submanifolds with symmetries, which had been defined only in finite dimensional Riemannian manifolds, into a class of proper Fredholm submanifolds in Hilbert spaces. Then I showed that there exist many infinite dimensional minimal submanifolds with symmetries. Consequently a critical difference between finite and infinite dimensional submanifolds became clear.

2. Backgrounds and problems

A *proper Fredholm submanifold* (PF submanifold), introduced by C.-L. Terng, is a Hilbert manifold immersed with finite codimension in a Hilbert space whose shape operators are compact operators and distance functions satisfies Palais-Smale condition. In the study of PF submanifolds an important tool is given by a certain Riemannian submersion $\Phi : V \to G/K$ which is called the *parallel transport map*. Here G/K is a compact normal homogeneous space and $V := L^2([0,1], \mathfrak{g})$ the Hilbert space of L^2-paths with values in the Lie algebra $\mathfrak{g}$ of G. It is known that for a closed submanifold N of G/K its inverse image $\Phi^{-1}(N)$ is a PF submanifold of V. It is an important problem to study the geometrical relation between N and $\Phi^{-1}(N)$.

In finite dimensional Riemannian manifolds there are several kinds of minimal submanifolds with certain symmetries, namely reflective submanifolds (D. S. Leung), weakly reflective submanfiolds (Ikawa-Sakai-Tasaki), austere submanifolds (Harvey-Lawson) and arid submanifolds (Taketomi). It is an interesting problem for a given isometric action to determine its minimal orbits with certain symmetries.

3. Main results ([2], [3] and [4] in the list of papers)

(a). *Submanifolds geometry of the parallel transport map.* I gave explicit Lie algebraic formulas for the second fundamental form and the shape operator of the PF submanifold $\Phi^{-1}(N)$. Using them I gave criteria for $\Phi^{-1}(N)$ to be totally geodesic. Moreover I gave an explicit formula for principal curvatures of $\Phi^{-1}(N)$.

(b). *Symmetric properties for the parallel transport map.* I defined and studied reflective PF submanfiolds, weakly reflective PF submanifolds, austere PF submanifolds and arid PF submanifolds in Hilbert spaces. Then under suitable assumption I showed that if N is a weakly reflective (resp. arid) submanifold of G/K then the PF submanifold $\Phi^{-1}(N)$ is also weakly reflective (resp. arid). Moreover I showed that if G/K is a sphere then the austerity of N is equivalent to the austerity of $\Phi^{-1}(N)$. Using these results I constructed many examples of weakly reflective PF submanifolds, austere PF submanifolds and arid PF submanifolds in Hilbert spaces.

(c). *Homogeneous minimal submanifolds in Hilbert spaces.* Combining results (a) with (b) it became clear that in infinite dimensional Hilbert spaces there exist many homogeneous minimal PF submanifold which are *not* totally geodesic. This shows a critical difference between finite and infinite dimensional submanifolds because all homogeneous minimal submanifolds in finite dimensional Euclidean spaces are known to be totally geodesic (Di Scala).