Results of my research

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A genus g handlebody-knot is a genus g handlebody embedded in the 3-sphere, denoted by H. Two handlebody-knots are equivalent if one can be transformed into the other by an isotopy of S^3. Cutting along a meridian disk system of H, we have a knotted solid torus in S^3. A constituent knot of H is the knot which is the spine of the knotted solid torus. The constituent knot depend on choice of a meridian disk. There are infinite many meridian disks for a handlebody-knot. Thus, there are many constituent knots for a handlebody-knot. Let $CK(H)$ be the set of all of constituent knots of H.

Last year, we have an necessary condition that a knot is a constituent knot of the handlebody-knot 4_1 by using Litherland's Alexander polynomial for handlebody-knots [2]. Here, 4_1 is the handlebody-knot in the table of genus 2 handlebody-knots with up to six crossings in [1].

Let $Conj(G_1, G_2)$ be the set of conjugacy classes of homomorphisms from G_1 to G_2. Let $G(K)$ be the knot group of a knot K. Let $\Delta_K(t)$ be the Alexander polynomial of K. We have the following theorem.

Theorem 1 [O.]
$K \in CK(4_1) \Rightarrow \#Conj(G(K), SL(2, \mathbb{Z}_3)) \leq 17$ or $\Delta_K(t)$ is reducible.

We have that the knot 10_{115} is not a constituent knot of the handlebody-knot 4_1 by Theorem 1.

References

[1] A. Ishii, K. Kishimoto, H. Moriuchi, and M. Suzuki, *A table of genus two handlebody-knots up to six crossings*, Journal of Knot Theory Ramifications **21**, No. 4, (2012) 1250035, 9 pp.

[2] S. Okazaki, *Litherland's Alexander polynomial for handlebody-knots*, preprint.