Summary of the study (Yosuke Saito)

The author studies mathematics related to the conformal field theory. Especially, the author is interested in a quantum many-body system called the Calogero-Moser system, and also interested in the Ruijsenaars system which is a q-deformation of the Calogero-Moser system. There are three types of the Calogero-Moser system called rational, trigonometric, and elliptic. The namings are used in the Ruijsenaars system in a similar way. The trigonometric Ruijsenaars system is understood very well by the theory of the Macdonald polynomials. In contrast, the elliptic Ruijsenaars system is difficult to study because of the appearance of elliptic functions. The following fact on the elliptic Calogero-Moser system is known: Let $H^{\mathrm{CM}}(\tau)$ be the Hamiltonian of the elliptic Calogero-Moser system. Then there exists a function $\Psi(\tau)$ satisfying

$$H^{\mathrm{CM}}(\tau)\Psi(\tau)=\frac{\partial\Psi}{\partial\tau}(\tau).$$

Where $\tau\in\mathbb{C}$ is an elliptic modulus which characterizes the system. The appearance of the τ derivative $\frac{\partial\Psi}{\partial\tau}(\tau)$ means that an infinitesimal deformation of the elliptic modulus arises in the elliptic Calogero-Moser system. Recalling that the elliptic Ruijsenaars system is a q-deformation of the elliptic Calogero-Moser system, the followings can be predicted: Let $H^{\mathrm{R}}(\tau)$ be the Hamiltonian of the elliptic Ruijsenaars system. Then there exist an operator which generates an difference deformation of the elliptic modulus in some sense and a function $\Psi(\tau)$ satisfying

$$\begin{aligned}&H^{\mathrm{R}}(\tau)\Psi(\tau)\\ =&(\text{an operator which generates a difference deformation of } \tau)\Psi(\tau).\end{aligned}$$