Result

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A field of my research is on invariants of 3-manifolds including finite type invariants of 3-manifold inspired by Witten's work on the Chern-Simons quantum field theory. In particular, I study the Chern-Simons perturbation theory developed by Kontsevich, Axelrod and Singer. The Chern-Simons perturbation theory gives sequences of invariants of 3-manifolds with acyclic local systems. Kuperberg and Thurston established a purely topological construction of the Chern-Simons perturbation theory at the trivial local system. There are deeply studies of the invariants by them, Taubes and Lescop. In particular, it is known that the Chern-Simons perturbation theory at the trivial local system gives a universal finite type invariant of integral homology 3-spheres. Although, the Chern-Simons perturbation theory at non-trivial local systems are not enough studied. A purely topological construction was given by Bott and Cattaneo. But there was a gap in their original construction. A purpose of my research is to establish and understand the non-trivial local system cases and to compare with the trivial local system case.

(1) I proved in [1] that the Chern-Simons perturbation theory at the trivial local system coincides with the Morse homotopy defined by T. Watanabe and K. Fukaya.

(2) "2-loop term" (or "Θ-term") is a part of the Chern-Simons perturbation theory related to the Θ-graph. In [2], I removed the gap in the construction of the Bott-Cattaneo construction of the 2-loop term for SU(2)-Chern-Simons perturbation theory. I also gave a corresponding Morse homotopy.

(3) (joint work with Alberto S. Cattaneo) We revisited the Bott-Cattaneo construction of the Chern-Simons perturbation theory and removed the gap. Our construction is a generalization of both the Bott-Cattaneo original construction and (2).

References

[1] Tatsuro Shimizu, An invariant of rational homology 3--spheres via vector fields, Algebraic and Geometric Topology, 16 (2016) pp.3073-3101
[2] Tatsuro Shimizu, Morse homotopy for the SU(2)-Chern--Simons perturbation theory, preprint, 2016
[3] Alberto S. Cattaneo and Tatsuro Shimizu, A note on the Θ-invariant of 3-manifolds, arXiv:1903.04386, 2019