Research Results

My main research area is differential geometry, and its keywords are "special Lagrangian submanifold" and "mean curvature flow". These two are important subjects of mirror symmetry. A special Lagrangian submanifold is defined as a minimal Lagrangian submanifold in a given Calabi-Yau manifold, and it can be written as a solution of a nonlinear elliptic PDE locally. Hence it is difficult to construct nontrivial concrete global examples of special Lagrangian submanifolds in a given Calabi-Yau manifold. For instance, in $\mathbb{C}^m$, there are previously known examples constructed by Harvey-Lawson and Joyce. In [Y, Special Lagrangians.., New York J. Math. 2016], I generalized their constructions in the cone of a toric Sasaki manifold.

Lagrangian mean curvature flows are important to obtain special Lagrangian submanifolds abstractly. Since the volume of a submanifold is decreasing along a mean curvature flow, the Lagrangian mean curvature flow might converge to a special Lagrangian submanifold if it has a long time solution with bounded curvature. In [Y, Weighted Hamiltonian.., Tohoku Math. J. 2016], I generalized the example constructed by Lee-Wang in $\mathbb{C}^m$ in toric almost Calabi-Yau manifolds. These examples of Lagrangian mean curvature flows develop singularities finitely many times and its topologies change when singularities occur.

As indicated by the above examples, Lagrangian mean curvature flows develop singularities, in general. Hence, what we should do is to analyze the asymtotic behavior of a Lagrangian mean curvature flow when it develops singularities. On this problem, in the case where the ambient space is $\mathbb{R}^m$, there is a well-known result due to Huisken, and Futaki, Hattori and I generalized his result in the case where the ambient space is a Riemannian cone manifold in [FHY, Self-similar solutions.., Osaka J. Math. 2014]. The main theorem states that if a mean curvature flow develops singularities of type I then its parabolic rescaling converges to a self-shrinker. In [Y, Ricci-mean curvature.., Asian J. Math.to appear], I further generalized these results to a coupled flow of a Ricci flow (constructed by a gradient shrinking Ricci soliton) and a mean curvature flow.

There are many known results about self-similar solutions in $\mathbb{R}^m$. Then, as natural interest, we can check that which results for self-similar solutions in $\mathbb{R}^m$ also hold for generalized self-similar solutions in gradient shrinking Ricci solitons in the sense of [Y, Ricci-mean curvature.., Asian J. Math.to appear]. Then, I generalized a result of Futaki-Li-Li which gives a lower bound of the diameter of a self-similar solution in $\mathbb{R}^m$ in [Y, Lagrangian self-similar.., J. Geom. 2017]. In this paper, I also generalized a result of Cao-Li.

In [Y, Ricci-mean curvature.., Asian J. Math.to appear], I considered a coupled flow of a Ricci flow and a mean curvature flow. It is called a Ricci-mean curvature flow. In [Y, Examples of Ricci-mean.., J. Geom. Anal. 2018], I constructed examples of Ricci mean curvature flows. Its ambient space is a gradient shrinking Kähler Ricci soliton on a total space of a $\mathbb{P}^1$-fibration over a projective space given by Cao and Koiso. For this ambient space, I investigated the motion of a lens space under a Ricci-mean curvature flow, and proved that if the initial radius of the lens space is lager than a specific value then it collapses to ∞-section and if the initial radius is smaller than the value then it collapses to 0-section. This gives a first nontrivial example of Ricci-mean curvature flows. Moreover, I and Koike studied general properties of Ricci-mean curvature flows, and in [KY, Gauss maps of.., Geom. Dedicata. 2018] we generalized a result due to M.-T. Wang (2003) to Ricci-mean curvature flow. Wang's results says that the Gauss map of a mean curvature flow in a Euclidean space is a harmonic map heat flow. We proved that this statement also holds for Ricci-mean curvature flows.

The theme of a paper [Y, Special Lagrangian.., Math. Z. 2018] is different from studies introduced above. In 2000, Leung–Yau–Zaslow proved that the special Lagrangian equation in a Calabi–Yau manifold X is converted into the deformed Hermitian Yang-Mills equations in a mirror Calabi–Yau manifold W when the base manifold B of X and W is an open set in a Euclidean space. In this paper, I generalized their correspondence in the case where B is a tropical manifold.

In [TY, Solutions with time-dependent.., J. Differential Equations. 2019], I and Takahashi studied singular solutions of the heat equation with absorption. The subject of this work is purely partial differential equations. The domain of the singular solution is a Euclidean space except a 1-parameter family of submanifolds. We studied the existence and nonexistence of singular solutions and asymptotic behavior of the solution near the singular set.

In [HY, An ε-regularity theorem.., arXix], I and Han studied line bundle mean curvature flows. The notion of line bundle mean curvature flows was recently introduced by Jacob and Yau as a kind of parabolic PDE to construct deformed Hermitian Yang–Mills metrics. If the flow has the long-time solution and converges to a metric when time tends to infinity, then the limiting metric should be a deformed Hermitian Yang–Mills metric. However, it is not known, so far, that the flow has the long-time solution in general. In this paper, we proved an ε-regularity theorem for line bundle mean curvature flows which says that if an analog of a Gauss density is smaller than $1 + \varepsilon$ then we get apriori estimates of the solution up to third derivatives.