International Workshop on
Differential Geometric Aspects of Integrable Systems

Kobe University

July 18–19, 2017

Organizers

Wayne Rossman (Kobe University, chair)
Kentaro Saji (Kobe University)
Yoshihiro Ohnita (Osaka City University)
Masashi Yasumoto (Osaka City University Advanced Mathematical Institute, co-chair)

Support

Research Project: "Geometric Aspects of Mathematics" (PI: Wayne Rossman)
Organization for Advanced and Integrated Research, Kobe University

Program

July 18th, 2017

13:00-13:05 **Opening**

13:05-13:50 Masatoshi Kokubu (Tokyo Denki University)

Kenmotsu-type formulas – Unification and Application

14:05-14:50 Mason Pember (Technische Universität Wien)

Deformability and integrable systems

14:50-16:15 **Poster Session & Tea Break**

Speakers of Poster Asahi Tsuchida, Joseph Cho, Yumiko Takei, Masahiro Morimoto, Mitsugu Abe, Hyeongki Park

16:15-17:00 Wai Yeung Lam (Brown University)

Minimal surfaces from deformations of circle packings

17:15-18:00 Kosuke Naokawa (Hiroshima Institute of Technology)

TBA

July 19th, 2017

10:00-10:45 Atsufumi Honda (Yokohama National University)

Isometric immersions with singularities between space forms of the same non-negative curvature

11:00-11:45 Shoichi Fujimori (Okayama University)

Quadrics and their Christoffel duals

13:00- Free Discussion

Titles & Abstracts

Title: Kenmotsu-type formulas – Unification and Application

Masatoshi Kokubu (Tokyo Denki University, Japan)

Abstract

TBA

Title: Deformability and integrable systems

Mason Pember (Technische Universität Wien, Austria)

Abstract

In this talk we shall see that G-deformability of maps in projective space is determined by the existence of certain Lie-algebra valued 1-forms. This result can then be applied to various examples, e.g., isothermic surfaces, to recover their well known gauge-theoretic approaches. We shall then hypothesise how this result may be applied to the more general setting of R-spaces.

Title: Minimal surfaces from deformations of circle packings

Wai Yeung Lam (Brown University, USA)

Abstract

We introduce discrete holomorphic quadratic differentials which relate infinitesimal deformations of circle packings to discrete minimal surfaces. Circle packings were proposed by Thurston to approximate holomorphic functions. We show that deformations of circle packings can be described in terms of vertex rotation. Under infinitesimal deformations, these rotation factors are harmonic in the sense of the cotangent Laplacian and the change in cross ratios lead to discrete holomorphic quadratic differentials. Via a Weierstrass representation, each holomorphic quadratic differential yields a polyhedral surface with vanishing mean curvature. Its relation to the integrable systems approach will be discussed.

Title: TBA

Kosuke Naokawa (Hiroshima Institute of Technology, Japan)

Abstract

TBA

Title: Isometric immersions with singularities between space forms of the same non-negative curvature

Atsufumi Honda (Yokohama National University, Japan)

Abstract

By the Hartman-Nirenberg theorem, any isometric immersion between flat space forms of codimension 1, i.e., complete flat hypersurface in Euclidean space, is a cylinder over a plane curve. However, there are many non-trivial flat surfaces with singularities called 'flat fronts'. Murata-Umehara classified complete flat fronts in Euclidean 3-space and proved their global properties, including the four vertex-type theorem. In this talk, we show the non-existence theorem of complete flat fronts in Euclidean space whose dimension is greater than three. Moreover, we classify isometric immersions with singularities (as wave fronts) between space forms of the same positive curvature, which is a generalization of a theorem of O'Neill and Stiel: any isometric immersion of the n-sphere into the $(n+1)$-sphere of the same sectional curvature is totally geodesic.

Title: Quadrics and their Christoffel duals

Shoichi Fujimori (Okayama University, Japan)

Abstract

We investigate the relation between quadrics and their Christoffel duals. It is known that the Christoffel dual of an ellipsoid is the affine image of a minimal surface, the Scherk tower, in Euclidean 3-space. If we think about the Christoffel dual of a one or two sheeted hyperboloid, then we have an affine image of certain zero mean curvature surface in Lorentz-Minkowski 3-space. This is a joint work with Udo Hertrich-Jeromin, Masatoshi Kokubu, Masaaki Umehara, and Kotaro Yamada.

Titles & Abstracts of Poster Presentation

Remark. Each speaker of the poster presentation gives a 5-minutes oral presentation about their own poster.

Title: Singular points of bundle homomorphisms from a tangent distribution into a vector bundle of the same rank

Asahi Tsuchida (Hokkaido University, Japan)

Abstract

We consider bundle homomorphisms between tangent distributions and vector bundles of the same rank. We study the conditions for fundamental singularities when the bundle homomorphism is induced from a Morin map. When the tangent distribution is the contact structure, we characterize singularities of the bundle homomorphism by using the Hamilton vector fields.

Title: Same lobed multibubbletons via Bianchi-Bäcklund transformation

Joseph Cho (Kobe University, Japan)

Abstract

From a given surface of constant positive Gaussian curvature (CPC), one may use the well-known Bianchi-Bäcklund transformation to obtain a new CPC surface. Then using the fact that a constant mean curvature (CMC) surface is a parallel surface of a CPC surface, one may obtain a new CMC surface from a given one via this transformation. A famous example of such transformation is explained in the work by Sterling and Wente, where they considered successive Bianchi-Bäcklund transformations of a circular cylinder, called a bubbleton and multibubbletons, using the latter as examples of non-finite type solutions to the sinh-Gordon equation. In this presentation, we give a method of constructing same lobed multibubbletons via the classical method of Bianchi-Bäcklund transformation. Furthermore, we extend this result to the Minkowski 3-space for spacelike CMC surfaces and its analogues to bubbleton in Euclidean 3-space. This presentation is based on the joint work with Yuta Ogata (National Institute of Technology, Okinawa) and Mitsugu Abe (Kobe University).

Title: Exact WKB analysis for confluent hypergeometric differential equations in terms of the topological recursion

Yumiko Takei (Kobe University, Japan)

Abstract

Voros coefficients are important objects in exact WKB analysis to study global behaviors of solutions of differential equations. In this poster I report that the Voros coefficients for confluent hypergeometric differential equations are given by the generating functions of free energies defined in terms of Eynard and Orantin's topological recursion. From these results, concrete forms of the free energies for algebraic curves related to these equations are obtained.

Title: Elliptic operators and compact groups

Masahiro Morimoto (Osaka City University)

Abstract

Let G be a compact Lie group. A G-invariant differential operator on a compact G-manifold is said to be transversally elliptic if it is elliptic in the directions transversal to the G-orbits. In this talk, we first review the spectral properties of an elliptic operator. After that, we study the spectral properties of a transversally elliptic operator and compare them.

Title: Constant mean curvature surfaces in hyperbolic 3-space with curvature lines on horospheres

Mitsugu Abe (Kobe University, Japan)

Abstract

Following the work of Wente on minimal surfaces in hyperbolic 3-space with one family of curvature lines on spheres, we classify constant mean curvature surfaces in hyperbolic 3-space with one family of curvature lines on horospheres. In doing so, we investigate the relationship between such surfaces and constant mean curvature rotation surfaces, both analytically and geometrically.

Title: Explicit Formulas for Area-preserving Deformations of Planar Curves in the Equicentroaffine Geometry

Hyeongki Park (Kyushu University, Japan)

Abstract

We present a formulation of discrete dynamics of discrete plane curves in the equicentroaffine geometry which is characterized as an area-preserving deformation. The deformation is governed by the discrete Korteweg-de Vries (KdV) equation. We also construct explicit formulas for the discrete deformation as well as the continuous deformation of smooth curves, in terms of the τ function of the (discrete) KdV equation. In the construction, we use the correspondence to the isoperimetric deformation of plane curves in the Minkowski plane.