

# The 13th Graduate Student Workshop on Mathematics

July 30 – August 1, 2019

Room E408, Department of Mathematics, Osaka City University

## Abstracts

July 30 (Tue.)

**Masahiro Morimoto** (Osaka City University, D3)

d17sa001@uv.osaka-cu.ac.jp

### On weakly reflective PF submanifolds in Hilbert spaces

A weakly reflective submanifold is a minimal submanifold of a Riemannian manifold which has a certain symmetry at each point. In my talk I will introduce this notion into a class of proper Fredholm (PF) submanifolds in Hilbert spaces and show that there exist so many infinite dimensional weakly reflective PF submanifolds in Hilbert spaces. In particular each fiber of the parallel transport map is shown to be weakly reflective. These imply that in infinite dimensional Hilbert spaces there exist so many homogeneous minimal submanifolds which are not totally geodesic, unlike in the finite dimensional Euclidean case.

**Hoyoung Jeong** (Pusan National University, D4)

gojung6655@hanmail.net

### Space-like Hyperurfaces Having Zero Mean Curvature in $L^4$ with two metrics

Research about space-like hypersurfaces with zero mean curvature in Lorentzian space is one of good topics in differential geometry. In  $L^{n+1}$ , hypersurfaces having zero mean curvature that consist zero mean curvature with respect to the Euclidean metric contain conical minimal surface in  $\mathbb{R}^n$ . Due to the reason, we see what space-like hypersurfaces having the previous condition in  $L^4$ .

**Luis Pedro Castellanos Moscoso** (Osaka City University, D1)

caste3.1416@gmail.com

### A classification of left-invariant symplectic structures on some Lie groups

In geometry it is an important problem to study whether a given manifold admits some nice geometric structures. In the setting of Lie groups it is natural to ask about the existence of left-invariant structures. A *symplectic Lie group* is a Lie group  $G$  endowed with a left-invariant symplectic form  $\omega$  (i.e. a nondegenerate closed 2-form). There are many interesting results about the structure of symplectic Lie groups and considerable

classification efforts in low dimensions, but the general picture is far from complete. In this talk we develop a method to classify left-invariant structures on a given Lie group: we first study the “moduli space of left-invariant nondegenerate 2-forms” and then we search inside this moduli space for the 2-forms that are symplectic. Finally we apply the method to some concrete examples.

**Kiwoong Kwon** (Kyungpook National University, D1)

kkw7076@knu.ac.kr

### **Graded $t$ -Schreier Domain and Its Application to Semigroup Rings**

Let  $R$  be an integral domain and  $M$  a nontrivial cancellative torsion-free commutative monoid. If the group of  $t$ -invertible  $t$ -ideals of the Riesz interpolation property,  $R$  is called  $t$ -Schreier. Similarly,  $M$  is called  $t$ -Schreier provided that the group of  $t$ -invertible  $t$ -ideals of satisfies the Riesz interpolation property. We show that the following properties of a commutative monoid domain  $R[M]$  are equivalent: (1)  $R[M]$  is  $t$ -Schreier; (2) Both  $R$  and  $M$  are  $t$ -Schreier.

**Hyein Choi** (Pusan National University, M2)

chi4361@naver.com

### **Observation on the multiplicity graph of an association scheme**

For a finite group  $G$ , its character degree set  $\text{cd}(G) = \{\chi(1) \mid \chi \in \text{Irr}(G)\}$  provides a lot of information on the structure of  $G$ . There are known results about the *character degree graphs* related to the number of connected components and the maximal number of independent vertices.

We want to observe above results in terms of association schemes. Define the *multiplicity graph* of an association scheme whose vertex set is the set of all prime divisors of multiplicities and  $p$  and  $q$  are adjacent if and only if  $pq$  divides for some multiplicity. By investigating the multiplicity graphs of association schemes of small order, we observe similarities and differences between multiplicity graphs and character degree graphs. In this talk, we would like to summarize what we have studied on multiplicity graphs and character degree graphs.

**Jan Kim** (Pusan National University, D2)

jmthblog@naver.com

### **Non-residually finite groups as direct limits of relatively hyperbolic groups**

A group  $G$  is called *residually finite* if for every  $g \in G \setminus \{1\}$ , there is some finite group  $P$  and an epimorphism  $\psi : G \rightarrow P$  such that  $\psi(g) \neq 1$ .

One of the challenging problems in geometric group theory is whether every hyperbolic group is residually finite. The concept of a relatively hyperbolic group is an important generalization of the concept of a hyperbolic group. So taking this challenge in case of relatively hyperbolic groups, we constructed non-residually finite groups as direct limits of relatively hyperbolic groups.

**Byeorhi Kim** (Kyungpook National University, D4)

kbrdooly@naver.com

### **On relation between group extensions and quandle extensions of a group**

In 1980s, Joyce and Matveev showed that a group  $G$  can be seen as a quandle  $(G, \triangleleft_\zeta)$ . In fact, they defined a quandle operation  $\triangleleft_\varphi$  on  $G$  by using a group automorphism  $\varphi$  on  $G$ . In this talk, we show a relationship between group extensions of a group  $G$  and quandle extensions of the quandle  $(G, \triangleleft_\zeta)$ . In fact, there exists a group homomorphism from  $H_{\text{gp}}^2(G; A)$  to  $H_q^2((G, \triangleleft_\zeta); A)$ . Also, we show a relationship between quandle extensions of a quandle  $Q$  and quandle extensions of the inner automorphism group  $\text{Inn}(Q)$  of  $Q$ . Indeed, there exists a group homomorphism from  $H_q^2(Q; A)$  to  $H_q^2((\text{Inn}(Q), \triangleleft_\zeta); A)$ . This is a joint work with Yongju Bae and J. Scott Carter.

**Yuta Taniguchi** (Osaka City University, M1)

m19sa012@du.osaka-cu.ac.jp

### **Quandle Presentation**

A quandle operation is a binary operation which satisfies the three axioms corresponding to Reidemeister moves in knot theory. A pair of a set and a quandle operation is called a quandle. Quandles are similar to groups. A group can be presented by generators and relations, called a group presentation. In the case of a quandle, a quandle can be presented by generators and relations. In this talk, I will introduce the notions of a quandle and a quandle presentation.

**Minju Seo** (Pusan National University, D1)

tjalswn92@naver.com

### **Quandle coloring quivers of surface-links via marked graph diagrams**

In 2018, K. Cho and S. Nelson introduced the quandle coloring quiver of an oriented knot or link diagram, which is a quiver structure on the set of quandle colorings of a knot or link diagram. Also, they gave a new invariant, called the in-degree quandle quiver polynomial, from the quiver structure.

A surface-link is a smooth embedding of a surface in the 4-space  $\mathbb{R}^4$  or  $S^4$ . A surface-link can be presented by a marked graph diagram with specific condition, and a marked graph diagram is a generalization of a knot or link diagram. In this talk, we define a quiver structure on the set of quandle colorings of a marked graph diagram, and compute the in-degree quandle quiver polynomials of some marked graph diagrams.

**Fumichika Takamizo** (Osaka City University, D2)

humichika.takamizo@gmail.com

### **Finite $\beta$ -expansion and odometers**

Let  $\beta > 1$  and  $T : x \mapsto \beta x \pmod{1}$ . Using the transformation  $T$ , we get a  $\beta$ -expansion  $\sum_{i=1}^{\infty} x_i \beta^{-i}$ , where  $x_i = \lfloor \beta T^{i-1} x \rfloor$ . We can define odometers associated with  $\beta$ . We say that  $\beta$  satisfies property (F) if and only if each number in  $\mathbb{Z}[1/\beta] \cap [0, \infty)$  has a finite  $\beta$ -expansion. In my talk, we construct a transducer which computes associated odometers

in the case  $\beta$  is a cubic Pisot unit with property (F). Akiyama proved a cubic Pisot unit satisfies property (F) if and only if  $\beta$  is a root of one of the following polynomial with integer coefficients: (1)  $x^3 - ax^2 - bx - 1 = 0$  ( $a \geq b \geq 1$ ), (2)  $x^3 - ax^2 - 1 = 0$  ( $a \geq 1$ ), (3)  $x^3 - ax^2 + x - 1 = 0$  ( $a \geq 2$ ), (4)  $x^3 - ax^2 - (a + 1)x - 1 = 0$  ( $a \geq 0$ ). By using the transducer, we give another proof of Akiyama's theorem.

**Hiroataka Kai** (Osaka City University, M2)

m18sa006@yf.osaka-cu.ac.jp

### Diffusion process on manifolds

Today, I will talk about how to construct diffusion process on Riemannian manifolds via orthonormal frame bundle and about some related theorems. I will show in three steps.

First step: construct a diffusion process on Euclidian (flat) space.

Second step: lift it to the frame bundle of the manifold.

Final step: project the frame bundle-valued diffusion process onto the manifold.

After the construction of manifold-valued diffusion process, I introduce the Bismut's formula

$$\nabla_x \log p_M(t, x, y) = \frac{1}{t} \mathbb{E}_{x,y,t} \left[ \int_0^t M_s dW_s \right],$$

which is a representation of logarithmic derivative of the heat kernel  $p_M(t, x, y)$ .

**Ryoya Tamaki** (Osaka City University, M1)

m19sa014@ka.osaka-cu.ac.jp

### Kolmogorov's zero-one law from a game-theoretic viewpoint

A game-theoretic framework is introduced by G. Shafer and V. Vovk in 2001. They and A. Takemura proved game-theoretic generalizations of several well-known zero-one laws in "The generality of the zero-one laws" (2011).

I explain how game-theoretic Kolmogorov's zero-one law can be shown by using the key concept "strategy" in this article.

**July 31 (Wed.)**

**Naoki Hamamoto** (Osaka City University, D6)

yhjyoe@yahoo.co.jp

### Sharp Hardy-Leray and Rellich-Leray inequalities for solenoidal fields

We show the best constant of Hardy-Leray inequality for solenoidal fields, together with its second order version called Rellich-Leray inequality. The solenoidal condition gives an improvement of the classical Hardy or Rellich inequality.

**Jinsoo Kim** (Kyungpook National University, M2)

ngfri@naver.com

### **Extreme symmetric 4-linear forms on the plane with the supremum norm**

The classification problems of the extreme points of the unit balls of the spaces of continuous (symmetric) multilinear forms and continuous (homogeneous) polynomials defined on a Banach space have been investigated since 1998. In this paper, we classify the extreme points of the unit ball of the Banach space consisting of symmetric 4-linear forms on, where denote the plane with the supremum norm. First, we obtain some exact formula for the norm of a given symmetric 4-linear form in. Using this formula, all the extreme points of the unit ball of were classified as follows:

$$\begin{aligned} \text{ext}B_{\mathcal{L}_s(4l_{\infty}^2)} = & \left\{ \pm (1, 0, 0, 0, 0), \pm \left(\frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right), \pm \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}\right), \right. \\ & \pm \left(\frac{1}{4}, -\frac{3}{4}, 0, \frac{1}{4}, 0\right), \pm \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}\right), \pm \left(\frac{7}{8}, -\frac{1}{8}, \frac{1}{8}, -\frac{1}{8}, \frac{1}{8}\right), \\ & \pm \left(\frac{3}{8}, \frac{3}{8}, -\frac{3}{8}, -\frac{1}{8}, \frac{1}{8}\right), \pm \left(\frac{1}{8}, -\frac{7}{8}, -\frac{1}{8}, \frac{1}{8}, -\frac{1}{8}\right), \pm \left(\frac{3}{8}, \frac{3}{8}, \frac{1}{8}, -\frac{1}{8}, -\frac{3}{8}\right), \\ & \pm \left(\frac{3}{4}, -\frac{1}{4}, 0, -\frac{1}{4}, 0\right), \pm \left(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right), \pm (0, 1, 0, 0, 0), \\ & \pm \left(\frac{3}{8}, \frac{3}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{3}{8}\right), \pm \left(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right), \pm \left(\frac{1}{8}, -\frac{7}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right), \\ & \left. \pm \left(\frac{3}{8}, \frac{3}{8}, \frac{3}{8}, -\frac{1}{8}, -\frac{1}{8}\right) \right\}. \end{aligned}$$

**Teruya Matsuura** (Osaka City University, M2)

476t700@gmail.com

### **Boundary blowing-up solutions to some semilinear elliptic equations**

In this talk, we provide exact boundary blowing-up solutions to some semilinear elliptic equations. Also we introduce an existence result for boundary blowing-up solutions for  $\Delta u = u^p$ .

**Sadaf Habibi** (Osaka City University, M2)

habibzaisadaf@gmail.com

### **A priori estimates for singular radial solutions to some semilinear elliptic equations with super critical growth**

We study singular radial solutions to some semilinear elliptic equations with super critical growth. We provide exact singular solutions and prove a priori estimates for any singular radial solution near the origin.

**David Iván Hernández Granados**

(Instituto Potosino de Investigación Científica y Tecnológica, D2)

matedavid.hg@gmail.com

### **Using graphs to the characterization of lncRNAs**

In biology, ribonucleic acid (RNA) is very important. In the two last decades, its study has grown significantly, especially, with the discovering of long noncoding RNAs (lncRNAs), which controllers the genetic expression for diverse cellular pathways. Likewise, the opportunities to use mathematics in diverse areas have had notable increment, for example, in the study of biological systems. At present, it is necessary to develop mathematical tools and data analysis that allow us to understand similarities and difference in some biological processes. An approach to studying the RNA is through the graphs use since the RNA can bend over itself to form segments bonded to hydrogens in different ways. This folding is known as the secondary structure of RNA. Representing this secondary structure through graphs has been very useful in the structural analysis to RNA, these models permit to describe the modular recurrent units of the RNA and its folding space. Thus in this talk, we will focus on the analysis of some chains of lncRNAs, by using graph theory, looking to extract structural information. This is a joint work with Hugo Cabrera Ibarra and Lina Riego Ruíz.

**Kazuaki Ogiwara** (Osaka City University, M2)

m18sa005@du.osaka-cu.ac.jp

### **Smooth torus orbit closures in the Grassmannians**

It is known that for the natural algebraic torus actions on the Grassmannians, the closures of torus orbits are toric varieties, and that these toric varieties are smooth if and only if the corresponding matroid polytopes are simple. In this talk, I will explain a result based on a joint work with Masashi Noji: simple matroid polytopes are products of simplices, and thus smooth torus orbit closures in the Grassmannians are products of complex projective spaces. If time permits, I will explain other results about subvarieties in the Grassmannians defined by matroids.

**Sanghoon Park** (Pusan National University, M2)

sanghoon1204@naver.com

### **An introduction to persistent homology**

Topological data analysis (TDA) is a branch of applied mathematics which studies the shape of data by using techniques from topology. As the main tool of TDA, persistent homology has been applied in wide areas such as biology and chemistry, astrophysics, automatic classification of images, sensor and social network analysis. In this talk, we review the basic concept of persistence and persistent homology group and introduce an algorithm for computing Betti numbers and persistence.

**Yonghyeon Jeon** (Kyungpook National University, D4)

dydgus1020@naver.com

### **Numerical Comparison of one-step method and multi-step method**

In this study, one-step method and a multi-stage method are numerically compared for stiff initial value problems. Traditionally, to avoid expensive computational costs required for solving a massive linear system induced from the linearization of a highly stiff system, stiff problems are usually solved by the multi-step method, rather than the

one-step method. We investigate a possibility of usage of multi-stage schemes for stiff systems by discussing the difference between the two schemes through several numerical experiments. Furthermore, the advantages of one-step methods are heuristically presented even for nonlinear stiff systems throughout several numerical tests.

**Hyobeen Kim** (Kyungpook National University, M2)

`hkb1436@naver.com`

### **Notes on list signed $H$ -colouring**

The switch homomorphism problem  $\text{switch}(H)$  for a signed graph  $H$  is known to be polynomial time solvable if  $H$  is one of three easily identified types and is otherwise  $NP$ -complete. We present results towards a similar dichotomy classification for the list version of the problem.

**Yoseb Kang** (Kyungpook National University, D1)

`josephdytpq@knu.ac.kr`

### **Multistability and periodic attractors of a nonsmooth atopic dermatitis system**

In this paper, we study what makes nonsmooth atopic dermatitis system complicated. We provide 4 subsystems to explain the progress of the disease. These subsystems and their fixed points give us the boundary conditions from healthy to chronic damage state, based on genetic condition. Near the boundary lines, interesting phenomena like bistability happen. Nonsmoothness of this atopic dermatitis system generates periodic attractors.

**Ryoya Kai** (Osaka City University, M2)

`m18sa007@ka.osaka-cu.ac.jp`

### **Commensurability of non-arithmetic hyperbolic 3-manifolds**

Two hyperbolic 3-orbifolds with finite volume are commensurable if they have a common finite sheeted covering space. Commensurability is an equivalence relation. It is known that the invariant trace field is an invariant of commensurability class. However, Coulson, Goodman, Hodgson and Neumann gave an example of a pair of closed manifolds that have the same invariant trace field and are incommensurable. They used an arithmetic tool to prove incommensurability of them. On the other hand, Han Yoshida gave an alternative proof for it, using the volume ratio of the manifolds.

I carried out an experiment to find out examples of pairs of closed hyperbolic 3-manifolds that have the same invariant trace field and are incommensurable by Yoshida's method. I will talk about the method and the result of this experiment.

**Tamotsu Baseda** (Osaka City University, M1)

`m19sa019@dt.osaka-cu.ac.jp`

### **Tangle surgeries which preserve the $Q$ polynomial of links**

The Q polynomial is an invariant of the isotopy type of an unoriented link defined by Brandt, Lickorish, Millett, and Ho. There are operations that change links without changing their Jones or HOMFLYPT polynomials. We consider analogous operations for links which do not change the Q polynomial.

**Suhyeon Jeong** (Pusan National University, D3)

j00399501303@pusan.ac.kr

### **Psybrackets and Pseudoknots**

In 2010, Ryo Hanaki introduced the notion of a *pseudodiagram*. A pseudodiagram is a knot or link diagram where we ignore over/under information at some crossings of the diagram. This definition is motivated by applications in molecular biology such as modeling knotted DNA, where data often comes inconclusive with respect to which crossing it represents.

Allison Henrich, Rebecca Hoberg, Slavik Jablan, Lee Johnson, Elizabeth Minten, and Ljiljana Radvić extended this idea to a pseudoknot and pseudolink. A *pseudoknot* (or *pseudolink*) is an equivalence class of pseudodiagrams modulo pseudo Reidemeister moves.

In this talk, we would like to introduce a *psybracket* consisting of two maps  $\langle, \rangle_c$  and  $\langle, \rangle_p: X \times X \times X \rightarrow X$  satisfying some axioms derived from pseudo Reidemeister moves. By using this, we define an invariant, called the *psybracket counting invariant*, of pseudolinks. This is a joint work with Jieon Kim and Sam Nelson.