The 11th KOOK-TAPU Joint Seminar on Knots and Related Topics

July 30 – August 1, 2019

Room E408, Department of Mathematics, Osaka City University

Abstracts

July 30 (Tue.)

Sam Nelson (Claremont McKenna College)

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Quandle Cocycle Quivers

Given a finite quandle X, a set of quandle endomrophisms $S \subset Hom(X, X)$ and a quandle 2-cocycle ϕ, we construct a directed graph-valued invariant of knots and link we call the quandle cocyle quiver. From this invariant we derive several polynomial enhancements of the quandle counting invariant and quandle cocycle invariant. This is joint work with my Harvey Mudd College senior thesis student Karina Cho.

María de los Angeles Guevara Hernández
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On the dealternating number and the alternation number

Links can be divided into alternating and non-alternating depending on if they possess an alternating diagram or not. After the proof of the Tait flype conjecture on alternating links, it became an important question to ask how a non-alternating link is "close to" alternating links. The dealternating and alternation numbers, which are invariants introduced by C. Adams et al. and A. Kawauchi, respectively, can deal with this question. By definitions, for any link, its alternation number is less than or equal to its dealternating number. It is known that in general the equality does not hold. However, in general, it is not easy to show a gap between these invariants. In this talk, we will show some results regarding these invariants. In particular, for each pair of positive integers, we will construct infinitely many knots, which have dealternating and alternation numbers determined for these integers. Therefore, an arbitrary gap between the values of these invariants will be obtained.

Shin'ya Okazaki (Osaka City University Advanced Mathematical Institute)

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Symmetric group representation of knot group and Nakanishi index

The Nakanishi index of a knot is the minimum size among all square Alexander matrix of the knot. In this talk, we show that a lower bound of the Nakanishi index of a knot is obtained by the number of elements in the set of representative elements of conjugacy classes of homomorphisms from the knot group of the knot to the symmetric group of order 6.

Jieon Kim (Pusan National University)

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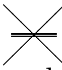
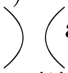
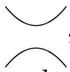
A bi-Gauss diagram of surface-links

M. Polyak and O. Viro introduced a Gauss diagram of classical links. A marked graph diagram is a diagram of a finite spatial regular graph with 4-valent rigid vertices such that each vertex has a marker. For a given marked graph diagram D, let $L_-(D)$ and $L_+(D)$ be classical link diagrams obtained from D by replacing each marked vertex  with  and , respectively. We call $L_-(D)$ and $L_+(D)$ the *negative resolution* and the *positive resolution* of D, respectively. Every surface-link can be represented by marked graph diagrams. In this paper, by using the Gauss diagrams of two resolutions of a marked graph diagram of a surface-link, we introduce a new method of describing surface-links, called a bi-Gauss diagram. This is a joint work with S. Bost, B. Garbuz and S. Nelson.

July 31 (Wed.)

Mark H. Siggers (Kyungpook National University)

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Discrete homotopy theory and the recolouring dichotomy

The H-recolouring problem, Recol(H), for a graph H, is the problem of deciding if one can move between two homomorphisms from a graph G to H via a sequence of homomorphisms in which we change the image of one vertex at a time. In the last 10 years, several papers have addressed the computational complexity of the problem Recol(H). We expect that Recol(H) will either be polynomial time solvable or PSPACE-complete, depending on H, and are trying to characterise which graphs H yield 'easy' problems. As Recol(H) can be rephrased as the problem of deciding if two maps are in the same component of the Hom-graph Hom(G, H)– a structure well known for applying topological ideas to graphs– it is not suprising that many papers on the topic use topological techniques. We describe the use of discrete homotopy theory in these papers, and consider to what extent homology can play in an eventual classification of the 'easy' graphs.

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New quantum invariants of IH-equivalent of genus 2 handlebody-knots

We defined new quantum invariants of IH-equivalent of genus 2 handlebody-knots. This invariant has a value which belongs to the quotient set obtained by dividing the complex number when a certain equivalence relation is decided. Thus, there exists a handlebody-knot that has a trivial value. Here, we talk about the usage of IH-equivalent and algebraic number theory. Then, we define a new quantum invariant of IH-equivalent of genus 2 handlebody-knots that has a value belonging to the ideal of the ring of integers. In addition, we introduce the proof of the invariant.

Hideo Takioka (Kobe University)

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4-move distance of knots

4-move is a local change for knots and links which changes 4 half twists to 0 half twists or vice versa. In 1979, Yasutaka Nakanishi conjectured that every knot can be changed by 4-moves to the trivial knot. This is still an open problem. In this talk, we consider 4-move distance of knots, which is the minimal number of 4-moves needed to deform one into the other. In particular, the 4-move unknotting number of a knot is the 4-move distance to the trivial knot. We give a table of the 4-move unknotting number of knots with up to 9 crossings. This is a joint work with Taizo Kanenobu.

Sang Youl Lee (Pusan National University)

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Shadow product biquandle cocycle invariants for surface-links

In 2001, J. S. Carter, S. Kamada and M. Saito introduced the shadow quandle cocycle invariants for classical links and surface-links (including more general cases) by using the shadow (co)homology theory of quandles, which are generalizations of quandle cocycle invariants introduced by J. S. Carter et al. These invariants for links and surface-links are defined as the state-sums of all Boltzman weights that are evaluations of a given 2- and 3-cocycle at the crossings of link diagrams and triple points of broken surface diagrams modulo Roseman moves, respectively, over all quandle colorings of arcs in link diagrams and sheets in broken surface diagrams together with particularly designed region (shadow) colorings for the complementary regions of the diagrams. Recently, those shadow quandle cocycle invariants for surface-links have been reinterpreted with biquandles using marked graph diagrams modulo Yoshikawa moves by S. Kamada, A. Kawauchi, J. Kim and S. Y. Lee. In this talk, I'd like to discuss some properties of the shadow product biquandle cocycle invariants for surface-links.

August 1 (Thu.)

Seung Yeop Yang (Kyungpook National University)

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On torsion in homology of finite connected quandles

It is known that the order of a finite quandle annihilates its reduced quandle homology groups and the torsion of its rack homology groups if the quandle is a quasigroup quandle(i.e. a latin quandle). If, however, a quandle is connected, the statement does not hold in general. We show that under certain conditions, the reduced quandle homology groups and the torsion of the rack homology groups of a finite connected quandle is annihilated by the order of the inner automorphism group of the quandle.

Kengo Kawamura (Osaka City University Advanced Mathematical Institute)

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A certain calculation of the Arf invariant of a proper link

An oriented link L is said to be proper if the linking number $\mathrm{lk}(K, L \setminus K)$ is even for each component K in L. The Arf invariant $\mathrm{Arf}(L) \in \{0, 1\}$ of a link L is defined only when L is a proper link. It is known that there are several ways to calculate $\mathrm{Arf}(L)$, e.g. using Seifert forms, the polynomial invariants, local moves, and 4-dimensional techniques. In this talk, we introduce an alternative way to calculate the Arf invariant of a proper link.