Group Invariant Solutions and Symmetric Criticality

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Lie's method of symmetry reduction for finding the group invariant solutions to partial differential equations is widely recognized as one of the most general and effective methods for obtaining exact solutions of non-linear partial differential equations. It is therefore quite surprising that Lie's method, as it is conventionally described, does not provide an appropriate theoretical framework for the derivation of such celebrated invariant solutions as the Schwarzschild solution of the vacuum Einstein equations, the instanton and monopole solutions in Yang-Mills theory or the Veronese map for the harmonic map equations.

I shall describe the elementary steps needed to correct this deficiency in the classical Lie method, and to give a precise formulation of the reduced differential equations for the group invariant solutions which arise from this generalization.

I will also discuss Palais' principle of symmetric criticality – this is the problem of determining those group actions for which the reduced equations of a system of Euler-Lagrange equations are derivable from a canonically defined Lagrangian. The *DifferentialGeometry* software will be used to give examples.