2. Future Study Plans

The applicant plans **to study relative numerical invariants for the (algebraic) fibered space $f : X \to B$, where B is a projective curve and general fibers of f are projective varieties of general type**. In what follows, we tacitly assume the above for fibered spaces. There exist important relative numerical invariants such as the self-intersection number K_f^n, the relative Euler number χ_f and the Hodge bundle $\deg f_*K_f$ for a fibered space $f : X \to B$ with $\dim(X) = n$. **Since fibered spaces are very diverse, their geometrical properties cannot be understood by a unified method**. Therefore, **the geography of fibered spaces** would be one of the basic study guidelines. The geography of fibered surfaces is a study of the relation between "geometrical properties of fibered surfaces" and "inequalities between relative numerical invariants" such as K_f^n, $\deg f_*K_f$ and χ_f. The geography of fibered surfaces has been at the center of the study of fibered surfaces. However, the geography of the fibered spaces with $\dim(X) \geq 3$ is still in its infancy, and even the foundation has not been established. The applicant's research plan is to "**establish a foundation for the geography of fibered spaces**" with general dimensions. Specifically, the following issues will be addressed.

1. On the triviality of fibered spaces

There exists no characterization by relative numerical invariants that fibered spaces are analytic fiber bundles. From the view of the deformation theory of complex structures, a fibered space is regarded as a deformation family of a projective variety. The question is whether it is possible to determine whether complex structures are deformed or not by depending only on relative numerical invariants. In the case of fibered surfaces, f being a fiber bundle and $\deg f_*K_f$ vanishing is equivalent. The applicant aims to find analogous results in three or more dimensions.

2. The region of existence of relative numerical invariants

The numerical range that relative numerical invariants can attain is called the region of existence of relative numerical invariants. It is known that the lower bound of K_f^n and $\deg f_*K_f$ is zero for fibered spaces with general dimensions. However, the lower bound of the relative Euler number χ_f is unknown. The explication of the optimal region is the basis of the geography of fibered spaces. For fibered surfaces, the relative Euler number χ_f coincides with $\deg f_*K_f$, hence its lower bound is zero. However, there exist 3-dimensional fibered spaces such that χ_f are negative. The applicant aims to find a lower bound of relative Euler numbers.

3. On the positivity of vector bundle f_*K_f

It is known that if a fibered space $f : X \to B$ is not an analytic fiber bundle, $f_*K_f^{\otimes m}$ is ample for a sufficiently large positive integer m. However, f_*K_f is not necessarily ample in general due to the influence of the free direct summand. The free direct summand, which is an obstruction to the ampleness, has not been studied at all in higher dimensions. For fibered surfaces, the rank of the free direct summand of f_*K_f increases the lower bound of the slope in many cases. However, it is unknown whether this phenomenon also occurs in three or more dimensions. For 3-dimensional fibered spaces, the applicant aims to clarify the effect of the free direct summand on the slope of f.