1. Summary of Previous Studies

(1) Back Ground

Let $f : S \to B$ be a surjective morphism from a complex smooth projective surface S to a smooth projective curve B with connected fibers. We call $f : S \to B$ a fibered surface. The applicant has studied fibered surfaces whose general fibers are of general type. In what follows, we tacitly assume that general fibers of fibered surfaces are of general type. For fibered surfaces, there exist important relative numerical invariants such as the square of the relative canonical bundle K_f^2 and the relative Euler number χ_f. Since fibered surfaces are very diverse, their geometrical properties cannot be understood by a unified method. Therefore, one of the basic study guidelines is **the geography of fibered surfaces**. In the geography of fibered surfaces, we study the relation between **geometrical properties of fibered surfaces** and **inequalities between relative numerical invariants**. For example, giving a lower bound of the slope K_f^2/χ_f for fibered surfaces with a certain geometric property is a fundamental problem in the geography of fibered surfaces. The applicant has studied the fibered surface admitting a cyclic covering of certain fibered surfaces (called primitive cyclic covering fibrations) from the view of geography and has written three papers.

(A1) **Slope inequalities for irregular cyclic covering fibrations.**

(A2) **Bounds for the order of automorphism groups of cyclic covering fibrations of a ruled surface.**

(A3) **Bounds for the order of automorphism groups of cyclic covering fibrations of an elliptic surface.**

(2) Summary of Papers

About (A1). There exists a still mysterious relative numerical invariant called **relative irregularity**. The relation between the "relative irregularity" and the "lower bound of the slope K_f^2/χ_f" has been studied extensively. In (A1), the applicant gives the lower bound of the slope of primitive cyclic covering fibrations with positive relative irregularity. Furthermore, the lower bound is optimal under some assumptions.

About (A2) and (A3). The relation between "geometric properties of fibered surfaces" and "upper bounds of the order of the automorphism groups" is a simple and interesting problem. More specifically, it is a question of whether the order of automorphism groups can be estimated by a function in "a constant depending only on a geometric property" and "relative numerical invariants". In (A2) and (A3), **the method of localizing the relative numerical invariants** is the axis. This method was used in the study of the slope inequality for primitive cyclic covering fibrations. In other words, **it is an application of geography**. In (A2), the applicant gives the upper bound of the automorphism groups for primitive cyclic covering fibrations of a ruled surface, which is a generalization of hyperelliptic fibrations. The upper bound is a function in the genus of general fibers, the genus of the base curve, the covering degree, and the square of the relative canonical bundle. In (A3), the applicant gives the upper bound of the automorphism groups for primitive cyclic covering fibrations of an elliptic surface, which is a generalization of bielliptic fibrations. A bielliptic fibration is a fibered surface admitting a double covering of an elliptic surface. The major difference between (A2) and (A3) is the method of analyzing the action of the automorphism on the singular fibers. Since elliptic surfaces have singular fibers unlike ruled surfaces, **the singular fibers of primitive cyclic covering fibrations of an elliptic surface are very complicated**. Therefore, the analysis of the action of the automorphism groups on singular fibers is very difficult. The applicant analyzed the action by using the meromorphic group structure of elliptic surfaces.