Summary of rsearch results to date

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Research to date can be classified into the following six issues.

(1) Determination of the structure of an artinian ring conditioned on indecomposable modules
(2) Studies on the first Tachikawa conjecture
(3) Studies on the derived equivalence classification of selfinjective algebras
(4) Covering theory for derived equivalences
(5) Construction of simple Lie algebras using Hall algebras of tame algebras
(6) Decomposition theory of modules (Applications to topological data analysis)

Each issue is explained below.

(1) Let A be an aritinian ring, J the Jacobson radical of A and $n \geq 1$. A right A-module M is called *n-th local* if $\operatorname{top}^n M := M/MJ^n$ is indecomposable, and A is said to be of right *n-th local type* if all indecopmpsable right A-modules are n-th local. When $n = 1$, M is local and A is of right local type in the usual sense. The structure of rings of right local type has been determined by Hiroyuki Tachikawa. In this task the structure of rings of second local type was mostly determined and fully determined in special cases. When A is a finite-dimensional algebra over an algebraically closed field, we characterized for A to be of this type by using quivers with relations.

(2) The Nakayama conjecture is true if and only if both the first and the second Tachikawa conjectures are true. In this task we investigated the first Tachikawa conjecture stating that a finite-dimensional algebra A over a filed k would be selfinjective if $\operatorname{Ext}^n_A(DA, A) = 0$ for all $n \geq 1$, where $D := \operatorname{Hom}_k(\cdot, k)$. When A is a commutative local algebra, we have shown that this holds without using assumptions for $n \geq 2$.

(3) For Brauer tree algebras, Rickard obtained complete invariants under derived equivalence and gave a derived equivalence classification. In our earlier work on this subject, we extended this result to the whole representation-finite self-incident algebras, determined the complete invariants under derived equivalences, and also gave the complete set of representatives under derived equivalences. This classification is further extended gradually to the class of selfinjective algebras, which are not necessarily representation-finite (including wild algebras).

(4) In this task, we aim to generalize the covering theory for derived equivalences, which played a decisive role in solving the above problem, to make it a more general and powerful tool. The main theorem can be written in the following form: When a group G is acting on two linear categories $\mathcal{C}$ and $\mathcal{C}'$, their orbit categories $\mathcal{C}/G$ and $\mathcal{C}'/G$ become derived equivalent if a tilting subcategory giving a derived equivalence between $\mathcal{C}$ and $\mathcal{C}'$ satisfies some extra conditions. This theorem could be extended to results in a much wider setting. Namely, we proved a similar theorem for the case where the orbit category construction of a group action on linear categories is extended to the Grothandieck construction of a lax (colax) action of a small category. This made it possible to construct new derived equivalences by glueing together many (even infinite) derived equivalences.

(5) For a simple Lie algebra, Ringel constructed the positive part by using the Hall algebra of a representaion-finite hereditary algebra. Xiao–Peng constructed the whole parts, but the parts were artificially pasted together, and the operations were also artificial and not constructed only by the Hall algebra operations. In this study, we constructed the whole parts of the simple Lie algebra by first consturcting a larger Lie algebra using the Hall algebra of a tame hereditary algebra (or a tame canonical algebra), and then by taking its factor algebra. In this way, the entire operation could be defined naturally by operations of the Hall algebra.

(6) This is a CREST research project to apply the representation theory of algebras to topological data analysis. From the viewpoint of the representation theory of algebras, this problem is to find the indecomposable decomposition of a given module. Using the Auslander-Reiten theory, we gave a general formula to compule the multiplicity of each indecomposable direct summand, and gave a concrete procedure for decomposing a given module using matrix problems for a special case of finite representation type.