RESEARCH PLAN

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We study the twisted Alexander polynomials associated to their holonomy representations of mutant knots with the same genus.

It is known that a pair of mutant knots shares the Alexander polynomial [1], the Jones polynomial [2], and the hyperbolic volume [3]. On the other hand, the genus of knots distinguish the mutant [4]. For example, Kinoshita-Terasaka knot and Conway knot are mutant knots with different genus, and we can distinguish these knots with the twisted Alexander polynomials [5]. However, there are some pairs of mutant knots with same genus, and we would like to study these knots.

According to [6], the twisted Alexander polynomial associated to the holonomy representation determines the genus for all hyperbolic knots with at most 15 crossings. Although the twisted Alexander polynomials associated to the holonomy representations can detect most pair of mutant knots up to 15 crossings, [6] shows that some pairs of mutant knots have the same twisted Alexander polynomials.

A sequence $a_1, \ldots, a_n$ of nonzero integers with common sign defines an alternating pretzel knot $P(a_1, \ldots, a_n)$, and the alternating pretzel knot $P(a_1, \ldots, a_j, \ldots, a_i, \ldots, a_n)$ obtained by switching some a_i and a_j $(i < j)$ is a mutant of the original $P(a_1, \ldots, a_n)$. For alternating knots, the genus is simply determined by the Alexander polynomial [7, 8]. Hence infinite pairs of pretzel knots described in the above are mutant knots with the same genus. We would like to compute the twisted Alexander polynomials associated to the holonomy representation for the family of these pretzel knots, and check if each pair have the same twisted Alexander polynomial. In general, it is difficult to obtain the holonomy representations of infinite family of knots. However, it is expected that we can obtain the holonomy representations by using the similar way as in [9].

References

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