Research plan

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In [Katsura and Takashima 2021], in the study of the isogeny based cryptography, a canonical form of the defining equation of a hyperelliptic curve of genus g which has an automorphism of order 2 is given. Under this condition, if g is odd, I will express the hyperelliptic functions of genus g in terms of those of genus $(g-1)/2$ and $(g+1)/2$, and if g is even, I will express the hyperelliptic functions of genus g in terms of two types of hyperelliptic functions of genus $g/2$. In [Shaska 2001], a canonical form of the defining equation of a hyperelliptic curve of genus 2 which admits a morphism of degree 3 to an elliptic curve is given. Under this condition, I will express the hyperelliptic functions of genus 2 in terms of the Weierstrass elliptic functions. It is well known that the hyperelliptic functions satisfy the KdV–equations and the KP–equations. By using the formulae to express the hyperelliptic functions in terms of the elliptic functions, it is possible to compute the values of the hyperelliptic functions by using the mathematical software such as Mathematica and Maple.

Let $f(x)$ be a polynomial of degree 5, V be the hyperellptic curve of genus 2 defined by $y^2 = f(x)$, and σ be the sigma function associated with V. For the vector of holomorphic one forms $\omega = {}^t\left(-\dfrac{xdx}{2y}, -\dfrac{dx}{2y}\right)$ on V, we consider the hyperelliptic integral $u = \displaystyle\int_{\infty}^{(x,y)} \omega$. In [Grant 1991] and [Buchstaber, Enolski, and Leykin 2012], meromorphic functions f_1 and f_2 on $\mathbb{C}^2$ are introduced, which are constructed by σ, it is shown that $y = f_1(u)$ and $y = f_2(u)$. Since the value of f_1 is equal to that of f_2 on the zero set of σ, by the general theory of functions of several complex variables, there exists a meromorphic function h on $\mathbb{C}^2$ such that $f_1 - f_2 = \sigma h$. I expressed h in terms of σ, which is a joint work with V. M. Buchstaber. On the other hand, in [Grant 1988] and [Matsutani 2003], meromorphic functions g_1 and g_2 on $\mathbb{C}^2$ are introduced, which are constructed by σ, it is shown that $x = g_1(u)$ and $x = g_2(u)$. Since the value of g_1 is equal to that of g_2 on the zero set of σ, by the general theory of functions of several complex variables, there exists a meromorphic function k on $\mathbb{C}^2$ such that $g_1 - g_2 = \sigma k$. I will consider the problem to express k in terms of σ. In theory of functions, it is an important problem to decompose a multivariable meromorphic function into a product of two meromorphic functions. I will give a non-trivial example for this problem.

For the hyperelliptic curves of genus $g = 2, 3$, I derived the partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the zero set of the sigma functions, which is a joint work with V. M. Buchstaber. These partial differential equations are two parametric deformations of the KdV–equations. The zero set of the sigma function associated with the hyperelliptic curve of genus g is equal to the image of $g-1$ points on the curve by the Abel–Jacobi map. For the hyperelliptic curves of genus g, I will derive partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the image of $k (< g)$ points on the curve by the Abel–Jacobi map. I think that these partial differential equations will be $2g - 2k$ parametric deformations of the KdV–equations.