Research project

Hidenori Fujiwara、

Researching the unitary representations of an exponential solvable Lie group G, the gap from nilpotent Lie groups is big. For example, if we induce two monomial representations from two real polarizations A, B satisfying the Pukanszky condition, they are unitary equivalent. When we consider a concrete problem in harmonic analysis, we sometimes need to know a explicit form of an intertwining operator between these two realizations. We easily get a formal intertwining operator, but the real problem is the convergence of the integral appearing there. A sufficient condition is that the simple product AB is a closed subset of G. I tackle this problem since a long time ago and I may recently arrive to a proof. I first check this proof and complete it.

A very useful tool in harmonic analysis of a nilpotent Lie group G is Casimir elements. These are elements of the enveloping algebra of its Lie algebra which give scalar operators under irreducible unitary representations of G. Corwin-Greenleaf showed their existence under some conditions on coadjoint orbits. I would like to consider whether we can extend or not their result to exponential solvable Lie groups. If we can show the existence of Casimir elements, they may be very useful for the study of the structure of the ring D(G/H) of the G-invariant differential operators associated to the monomial representation. For example, one may treat Duflo's problem asking whether the center of D(G/H) is isomorphic or not to the Poisson center of the algebra of H-invariant polynomial functions on a certain affine subspace of the dual vector space of the Lie algebra.

In more detail, let G be an exponential solvable Lie group with Lie algebra $\mathfrak{g}$, $f \in g^*$ and H a connected closed subgroup of G with Lie algebra $\mathfrak{h}$ verifying $f([\mathfrak{h}, \mathfrak{h}]) = \{0\}$. We construct the monomial representation τ of G induced from the unitary character $\chi_f(\exp X) = e^{if(X)}$ $(X \in \mathfrak{h})$ of H. Further we put $\Gamma = f + h^{\perp}$. Then, if τ has multiplicities of discrete type, we can describe its Plancherel formula and the ring D(G/H) is commutative. In this situation, is D(G/H) isomorphic to the ring of the H-invariant polynomial functions on Γ ? I would like to research these problems.