

Résumé of my research

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I started my mathematical study in early 1970's at master course of Tokyo University. I began my study of unitary representations and harmonic analysis of solvable Lie groups under Professor Masahiko Saito who had just returned from his second stay in France. At that time, following the creation of the orbit method by Kirillov, this field was very active. In particular, Dixmier, Duflo, Vergne and many young people around them made University of Paris one of the research centers of the world. I researched the orbit method for unitary representations of solvable Lie groups under the direction of Professor Duflo at University Paris 7 as scholar of French Government and in CNRS as researcher.

Different from semisimple Lie groups, solvable Lie groups are poor in algebraic structure and there are not any tools as root system or maximal compact subgroup. Hence we must rely on induction whose internal matter is ambiguous. Nevertheless, a beautiful result may occur. A useful tool for such result is the orbit method which realize the unitary dual of Lie groups by the space of coadjoint orbits.

Let G be an exponential solvable Lie group, namely its exponential map is a diffeomorphism from its Lie algebra onto G .

1. Let τ be a monomial representation of G , namely induced from a unitary character of a connected closed subgroup H . I gave the irreducible decomposition of τ in terms of the orbit method.
2. Let π be an irreducible unitary representation of G and K a connected closed subgroup of G . I gave the irreducible decomposition of the restriction of π to K in terms of the orbit method.

Let G be a connected and simply connected nilpotent Lie group and τ a monomial representation of G . Then, Corwin-Greenleaf presented the following two conjectures.

Commutativity conjecture : τ has finite multiplicities if and only if the ring $D(G/H)$ of the G -invariant differential operators on the line bundle associated to τ is commutative.

Polynomial conjecture : When τ has finite multiplicities, $D(G/H)$ is a polynomial ring.

3. I established these two conjectures and also two counterparts for restrictions.