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Construction of p-adic triple product L-functions attached to universal deformation rings

Let p be an odd prime. By [p382, Maz87], we can construct a family of Galois representations called a universal deformation ring for each modular form f. There exists a correspondence between Galois representations and modular forms. Then we can regard universal deformation rings as families of modular forms. Let F and G be Hida families. In [Fuk19], we constructed p-adic triple product L-functions attached to triples (F,G,H) where H were general p-adic families of modular forms which satisfied some conditions. In particular, we can take Hida families and Coleman families as examples of H. However in [Fuku19], we can not take universal deformation rings as an example of H. Hence we want to construct p-adic triple product L-functions attached to universal deformation rings. We can regard Hida families and Coleman families as a specialization of the universal deformation. If we can construct p-adic triple product L-functions attached to universal deformation rings, it is a more universal theory of p-adic triple product L-functions.

Since universal deformation rings are defined by families of Galois representations, it is difficult to get ℓ-th Fourier coefficients of p-adic families of modular forms attached to universal deformation rings for each prime ℓ. So we can not apply the proof of [Hsi17] literally and have to modify the proof to construct p-adic triple product L-functions attached to universal deformation rings. For each universal deformation ring, we define the tame level of the universal deformation ring to be the tame level of Galois representations attached to the universal deformation ring. Let H be a p-adic family of modular forms attached to a universal deformation ring with tame level N. It is difficult to get the ℓ-th Fourier coefficient of H for each prime $\ell \mid Np$. However, if (F,G,H) is a triple of Hida families, we do not need the p-th coefficient of H to construct p-adic triple product L-functions attached to (F,G,H). Then if H is a p-adic family of modular forms attached to a universal deformation ring with tame level 1, we can construct a p-adic triple product L-function attached to (F,G,H) where F and G are Hida families. Our goal is to construct p-adic triple product L-functions attached to two Hida families F,G and a p-adic family H of modular forms attached to a universal deformation ring with general tame level.

Explicit formula for central critical values of triple product L-functions with Gejima

We will construct explicit formulas for central critical values of triple product L-functions with Gejima If we get the explicit formula, we can calculate the explicit value of the central critical value of the triple product L-function for each triple of modular forms. To get the explicit formula, we have to get many explicit formulas for an average of central critical values of triple product L-functions attached to triples of modular forms with same level and same weight. After that, we solve the simultaneous equations of the explicit formulas of the average. Then, we can get the explicit formula for the central critical values of triple product L-functions.

Reference

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