RSEARCH PLAN

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Background and Overview

In the early 1990s, for automorphic forms F and f on SO_{n+1} and SO_n, respectively, Gross–Prasad conjectured that a perid $\langle F|_{SO_n}, f \rangle$ vanishes if and only if the central value of an automorphic L-function for F and f does. After that, Ichino–Ikeda refined their conjecture by writing down an explicit conjectural relationship between the period and the central value. It is natural to think of an analogue of Gross–Prasad conjecture and Ichino–Ikeda cojecture for automorphic forms on other groups. For example, the analogue of Gross–Prasad conjecture for classical groups was formulated by Gan–Gross–Prasad. On the other hand, analogues of Gross–Prasad conjecture and Ichino–Ikeda cojecture for *non*-reductive groups have been hardly investigated, as far as I know.

Current Research Projects

I'll sudy about an analogue of Ichino–Ikeda conjecture for Jacobi forms, which are one of the classical automorphic forms and related to Siegel automorphic forms.

To explain more precisely, I introduce notation of Jacobi groups. Let $\mathbb{A}$ be the adele ring of the rational number field $\mathbb{Q}$ and set

$$ G_n = \left\{ g \in GL_{2n} \mid {}^t g \begin{pmatrix} & I_n \\ -I_n & \end{pmatrix} g = \begin{pmatrix} & I_n \\ -I_n & \end{pmatrix} \right\}, $$

which is the syplectic group over $\mathbb{Q}$. Let H_n denote the Heisenberg group defined by $H_n(\mathbb{Q}) = \mathbb{Q}^n \times \mathbb{Q}^n \times \mathbb{Q}$. Here the multiplication of H_n is given by

$$ (u,v,t)(u',v',t') = \left(u+u', v+v', t+t' + \tfrac{{}^t u v' - {}^t v u'}{2} \right). $$

The group G_n acts on H_n naturaly. The semidirect group G_n^J of G_n and H_n is called the Jacobi group and automorfphic forms on $G_n^J(\mathbb{A})$ are called Jacobi forms. There are Hecke theories for Siegel automorphic forms and Jacobi forms. Let φ be a Hecke eigen Jacobi form on $G_n^J(\mathbb{A})$ and f a Hecke eigen Siegel automorphic form on $G_n(\mathbb{A})$. The restriction of φ to $G_n(\mathbb{A})$ is also a Siegel automorphic form on $G_n(\mathbb{A})$. Then the problem I want to study is as follows: Can we determine the vanishing of the Petersson inner product $\langle \varphi|_{G_n}, f \rangle$, which is called the period of φ and f, by the vanishing of a special value of an automorphic L-function for φ and f ? This is an analogue of Gross–Prasad conjecture. More precisely I will write down an explicit conjectural relationship between the period and the special value, formulate an analogue of Ichino–Ikeda conjectre for Jacobi forms and Siegel automorphic forms and try to solve it.