

## Research Plan

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The geometrical mapping method we worked on in our previous study [8] has proven to be a powerful tool for comprehensively understanding how dynamical systems change. We have shown that the shape of the factor graph of a mechanical word of order  $n$  with slope  $\alpha$  is entirely determined by the position of  $\alpha$  in the Farey sequence with the maximum denominator  $n + 1$  and that the relationship between the position of the prefix of length  $n$  of a mechanical word determined by the slope  $\alpha$  and the intercept  $\rho$ , and  $\alpha$  and  $\rho$  in the factor graph.

We will examine specifically how far the geometric mapping method can be applied. The Sturmian words are the first step in this direction.

Since Sturmian words are closely connected to various fields of mathematics, there are multiple directions in which the research can be taken. Specifically,

- Examination of the relationship with return words, derivated sequences [6], Sós permutations [5]
- Considering of increasing the number of letters in the alphabet [2]
- Loosening the height difference condition as a variation on the concept of balance
- Loosening the periodicity condition (quasiperiodicity) [3]
- Investigation of Ostrowski numeration system [4], [7]
- Numeration systems and tilings as one extension to higher dimensions [1]

can be considered. We will explore current research for further extensions and generalizations in these related areas.

## References

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