参考文献

[1] R. H. Bamler, Compactness theory of the space of Super Ricci flows, arXiv preprint arXiv:2008.09298 (2020).

[2] S. Hamanaka, Ricci flow with bounded curvature integrals, Pacific J. Math. **314** (2021), 283–309.

[3] S. Hamanaka, Type of finite time singularities of the Ricci flow with bounded scalar curvature, arXiv preprint arXiv:2105.08250 (2021).

[4] S. Hamanaka, C^0, C^1-limit theorems for total scalar curvatures, arXiv preprint arXiv:2208.01865 (2022).

[5] S. Hamanaka, Upper bound preservation of the total scalar curvature in a conformal class, in preparation.

[6] M. Llarull, Sharp estimates and the Dirac operator, Math. Ann. **310** (1998), 55–71.

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上記内容の英訳

I'm interested in the shape of a manifold endowed with a Riemannian metric structure, and the topology of some distinctive subspaces of the space of all Riemannian metrics, e.g., the space of positive scalar curvature metircs. Recently I'm mainly interested in scalar curvatures. The main method that I'm using is geometric analysis, especially Ricci flows. I'm also interested in Ricci flows themselves and study the behavior of them. My current research themes are as follows : **(a)** 「Analysis of finite-time singularities of Ricci flows on a closed manifold (RF for short) of dimension at least four」 and **(b)** 「Study of some distinctive subspace of the space of all Riemannian metrics on a (closed) manifold by using some geometric flow techniques」. Here are some more specific topics of my research :

(a)-1 As an extension of [2], I try to investigate the behavior as $t \to T$ of RF $(M, g(t))_{t\in[0,T)}$ $(T < \infty)$ whose L^2-energy of the scalar curvature is bounded. More specifically, I try to investigate that whether such flow can converge or not, in the sense of Gromov-Hausdorff or Metric flow (recently defined by Bamler [1]) as $t \to T$, and if do, what is the limiting space. Moreover, we also want to know the topology and some kind of dimensions of a singular set of such limiting space, and extendability of the flow over the time T.

(a)-2 In [3], I showed that the shape of certain type of singularities of RF $(M, g(t))_{t\in[0,T)}$ $(T < \infty)$, whose scalar curvature is bounded are restricted in some sense. I try to construct an example of RF whose scalar curvature is bounded with such singularities, and study the structure of such singularities simultaneously. For this, we firstly focus on the four-dimensional case.

(a)-3 In [4], I showed that a preservability of the lower bound of the total scalar curvature measured by a weighted Riemannian volume measure. To prove this, I used an observation of a short-time behavior of certain quantity along the Ricci flow and the heat flow coupled with the Ricci flow. Meanwhile, I want to study that if we flow certain good metric and function, how such flows behave in a suffuciently large time. I also consider an application of such flows to studying various weighted curvatures.

(b)-1 In [4], I proved the following. Let M^n be a smooth manifold of dimension $n \geq 2$ and $\mathcal{M}$ a space of all Riemannian metrics on M. For any nonnegative continuous function $\sigma \in C^0(M)$ and constant $\kappa \in \mathbb{R}$, the space

$$\left\{ g \in \mathcal{M} \,\middle|\, \int_M R(g)\, d\mathrm{vol}_g \geq \kappa,\ R(g) \geq \sigma \text{ on } M \right\}$$

is C^1-closed in $\mathcal{M}$. Here, $R(g)$ denotes the scalar curvature of a metric g. Thus, I try to investigate whether we can remove the condition "$R(g) \geq \sigma$" or not.

(b)-2 Related to the results in [4], I try to clarify some relations between the parallelizability and the behavior of the total scalar curvature with respect to C^1-topology on a closed manifold.

(b)-3 Related to the examples constructed in [4], I try to investigate whether or not we can construct an example of noncompact complete surface on which the main theorem does not hold. I also want to study some relations between the size of the set consisting of points on which metrics does not converge in C^2 sense and the behavior of the total scalar curvature on a closed manifold.

(b)-4 I try to investigate a total-scalar-curvature-version of the rigidity result of [6]. Moreover, I want to consider whether or not a similar rigidity result also holds in the case that the metric only has low regularity, which is weaker than C^2-topology.

(b)-5 In [5], I proved the following. Let $g_0 \in \mathcal{M}$. Then, for any continuous function σ and constant κ, when the conformal class $[g_0]$ of g_0 is Yamabe positive or nonpositive respectively,

$$\left\{ g \in [g_0] \;\middle|\; \int_M R(g)\, d\mathrm{vol}_g \leq \kappa,\ R(g) \geq \sigma \right\}, \quad \left\{ g \in [g_0] \;\middle|\; \int_M R(g)\, d\mathrm{vol}_g \leq \kappa \right\}$$

are closed in $\mathcal{M}$ with respect to C^0-topology respectively. (Here, we used the same notations as in (b)-1.) I proved this result by using a stability property of the Yamabe flow. On the other hand, a notion of weighted Yamabe flow has been studied. I also try to investigate the spaces defined by replacing the total scalar curvature in the spaces described above with a kind of weighted scalar curvature.