Masato Hashizume

I studied the following topics:

(1) L^p-Lyapunov inequalities,

(2) Minimization problems on Hardy-Sobolev inequality,

(3) On compactness of Sobolev embedding involving variable exponent,

(4) Variational problem and elliptic equations on Trudinger-Moser inequality.

In these topics, I introduce (2) and (4) in detail.

(2) Minimization problems on Hardy-Sobolev inequality

I studied the Neumann problem on the Hardy-Sobolev inequality. This inequality is related to the embedding from Sobolev spaces to the Lebesgue spaces with weighted function. By singularity of the weighted function and an invariance of a scaling of functions, the embedding is non-compact. Concerning this problem, some topological properties of bounded domain plays a important role. As the previous work, it is known that there is a minimizer when the singularity is located on the boundary and the mean curvature at the singularity is positive. I studied other cases and I showed that the scale of the domain is related to the existence and nonexistence of minimizer. Under some conditions, I clarified the necessary and sufficient condition on existence of a minimizer. Based on these results, I studied the related elliptic equation with C.-H. Hsia and G. Hwang. Recently, we studied an elliptic equation involving Hardy-Sobolev nonlinearity and multiple Hardy terms. We clarified the relationship between existence of a positive solution and existence of an extremal of corresponding variational problem.

(4) Variational problem and elliptic equations on Trudinger-Moser inequality

It is known that there exists a extremal for the classical Trudinger-Moser inequality. By Mancini-Martinazzi（2017）, it was shown that the asymptotic expansion of the Dirichlet energy of some concentrating sequence, which is strictly related to the variational problem. After their study, P.-D. Thizy (2018) and Ibrahim-Masmoudi-Nakanishi-Sani (2020) showed the threshold nonlinearity dividing existence and nonexistence of maximizer. At the same time, I also showed one of the threshold nonlinearity dividing existence and nonexistence of a maximizer and clarified the relationship between existence of a maximizer and a behavior of lower order perturbation in detail.

I studied positive critical points of the Trudinger-Moser functional. The uniqueness result and some asymptotic properties of the critical points were proved. Moreover, I proved asymptotic expansion of the best constant of the Trudinger-Moser inequality with respect to scale parameter. Using the asymptotic expansion, I proved that maximum point of maximizers for the Trudinger-Moser inequality is located on boundary of domain, and then the maximum point maximize the curvature on the boundary.

I also obtained several results on the relationship between the Sobolev inequality and the Trudinger-Moser inequality. This study is joint work with N. Ioku.