（英文）

（Qualitative Research） Fatou, in his study of holomorphic maps on the Riemann sphere, conjectured that hyperbolicity is a density property in the space of all holomorphic maps (1920). This conjecture was proved correct for real quadratic polynomials by Granczyk-Swiatek and Lyubich, and for higher order real polynomials by Kozlowski-Shen-van Strien. But it is not solved for the complex case yet. This researcher believes that by deepening the ideas of Mane-Sad-Sullivan, this conjecture could be proved. The proof was incomplete a few years ago, but is now nearly complete and a paper is in preparation. It will not take much time to complete the paper. The ideas obtained in this research will be used to promote research on complex dynamical systems. In the theory of differentiable dynamical systems, most of the problems raised by Smale and others in the 1960s are still open, and as a breakthrough to solve them, he will consider vector bundle automorphism on vector bundles with Finsler metric on a compact metric space, and will develop the geometry analogue of Oseledets' Multiplicative Ergodic Theorem. The goal of the project is to give impetus to the currently stalled research on differentiable dynamical systems and Pesin theory.

（Quantitative Research） The new methods described in the Summary of Research Results will be applied to the problem of singular perturbations in a variety of phenomena. As an example for simplicity, they will discuss what happnes in case $\varepsilon \to 0$ in the second-order nonlinear differential equation; $\varepsilon^2 \frac{d^2x}{dt^2} = x^2$. This problem has been described by physicists Voros, Hakkim, Tobis and Tokihiro Nakamura around 2000. The internal solution is considered to be a solution to the difference equation obtained by differencing the derivatives. The difference equation corresponds to a two-dimensional symplectic map and the solution gives two invariant curves. By showing that these curves intersect transversely, one can conclude that the internal solution of the phenomenon treated as a singular perturbation problem is chaotic. However, the solution is an entire function, the specific determination of which is still an open question, and the global picture of these curves is not known. The difference equation is simple. But, the quadratic equation when the linear part of the left-hand side in the difference equation is viewed as the operator has the double root one, and so it corresponds to a parabolic case. This is an insurmountable difficulty. To overcome this difficulty, they consider approximating it with hyperbolic ones. By renormalizing ε and introducing the variable $\tilde{s}$, the solution $x(t)$ of the difference equation would be obtained as follows; $x(t) + x(2t) + x(3t) + \cdots = \sum_{n=2}^{\infty} \frac{c_n}{n} \frac{1}{\tilde{s}^n}$, where the coefficients c_n would be determined by considering all higher-order differences. To solve this for $x(t)$, they would use the special value of the prime zeta function. In this scheme, they would obtain the concrete representation of the entire function in siries of $1/\tilde{s}$ and solve the difficult problem related to the resurgent analysis described above. Together with the hyperbolic case (the paper has been submitted), he will apply the methods to single pendulum, double pendulum, and recent dynamical tunneling problems, and conduct quantitative research in the field of applied mathematics using computer, in collaboration with Chihiro Matsuoka.