Future research plan

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I will study the following in analysis for random functions:

(1) Ogawa integrability

In [5], we showed that the Ogawa integral $\int_0^t X(s)\, d_\varphi B_s$ with respect to a Brownian motion B of a certain Skorokhod integral process X converges for each $t \in [0, L]$. In the causal case, [7] indicates that this convergence is uniform in $t \in [0, L]$, and therefore X satisfies the condition called regular integrability. Besides, [8] gave the unique existence of the stochastic integral equation by the regular integrable random function. So, I investigate the regular integrability of X in the general case. Also, I study the Ogawa integrability of the random function which is integrable with respect to the Nualart-Pardoux-Stratonovich integral (NPS integral for short) presented in [6, Theorem 7.3] and the Ogawa integrability in the case that the space $[0, L]$ of time parameter t is generalized.

(2) Riemann approximation of the stochastic integral

Introducing the regularized Riemann sum $R_\Delta(X; Y)$ for random functions X and Y and a weighted partition Δ of $[0, T]$, I constructed the stochastic integral $I_\alpha(X; Y) = \int X d_\alpha Y$ based on R_Δ indexed by a measurable function α from $[0, T]$ to $[0, 1]$ with the following two property ([3]):

(a) When Y is a Brownian motion B and $\alpha = \frac{1}{2}$, I_α approximates the Ogawa integral regarding regular CONSs and NPS integral.

(b) When Y is a continuous semi-martingale and $\alpha = \frac{1}{2}$, I_α approximates the Fisk-Stratonovich integral with respect to Y.

When Y is a Brownian motion, [7] shows the fundamental fact on the Ogawa integral that "every Brownian quasi-martingale is φ-integrable" is equivalent to "φ is regular" for a CONS φ which defines the Ogawa integral. On the analogy of this fact, when Y is a continuous quasi-martingale, I gave a necessary and sufficient condition for a weighted partition Δ to I_α exists for every quasi-martingale.

The Ogawa integral is defined with respect to a Brownian motion, while the integral I_α given in [3] is defined with respect to a quasi-martingale and includes as a special case the Ogawa integral regarding regular CONSs in the sense in (a). So, I want to answer the question whether the same fact holds for the integral I_α with respect to a (continuous or discontinuous) quasi-martingale as on the integrability and application given in [9] for the Ogawa integral.

(3) Stochastic differentiability, identification of random functions from the SFCs

The stochastic derivative and quadratic variation is the fundamental operations as the inverse of the stochastic integral. However the sum of random functions X and Y with the quadratic variations does not necessarily have the quadratic variation in general. In this study, giving a general vector space or an algebra of random functions with quadratic variations, we shall present that we can unifiedly calculate the stochastic derivative and quadratic variation as the inverse of stochastic integral independent of theory of stochastic integral.

I gave a theorem in [2], [4] roughly described as follows: for a random function V with the quadratic variation, a class

$$Q(V) = \left\{ X : \text{random function} \;\middle|\; [X],\ \frac{dX}{dV} \text{ exists and } \frac{d[X]}{d[V]} = \left|\frac{dX}{dV}\right|^2 \right\}$$

of random functions with the quadratic variation $[\,]$ and stochastic derivative $\frac{d}{dV}$ with respect to V forms a vector space. Here, this theorem is used to prove the result in [1] on the identification of random functions from the SFCs mentioned in "Summary of research results so far".

Next, we work on examining whether $Q(V)$ forms a ring and giving a larger linear space than $Q(V)$ as a class of random functions with the quadratic variations. Besides, we attempt to apply this study to the identification problem of random functions from the SFCs.

References

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