Summary of research results so far

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In analysis for random functions I gave results as follows:

(1) Ogawa integrability ([3])

[5] shows that the Ogawa integral of the Itô process is given with the Itô integral. I and Tetsuya Kazumi showed that the Ogawa integral of the S-type Itô process, which is a noncausal extension of the Itô process, is given with the Skorokhod integral and its adjoint operator.

(2) Identification of random functions from the SFCs ([1, 2, 4])

Let X_t, $t \in [0, L]$ be a random function $X_t = \int_0^t a(t)\,dB_t + \int_0^t b(t)\,dt$ driven by a Brownian motion B. To the question whether the coefficients $a(t)$ and $b(t)$ are determined by the stochastic Fourier coefficients (SFCs for short):

$$(e_n, dX) = \int_0^L e_n\,dX$$

with respect to a CONS $(e_n)_{n\in\mathbb{N}}$ of $L^2([0, L])$ posed in [6], we got the following affirmative answer, employing the SFCs in the case that $\int dB$ is the Skorokhod integral (SFC-Ss in abbr.) and the SFCs in the case that $\int dB$ is the Ogawa integral (SFC-Os in abbr.): here, by FVP, $\mathcal{L}^{r,2}$ we mean the totality of random functions of bounded variation, totality of square itegrable Wiener functionals with differentiability index r, respectively.

- Derivation of random functions from SFC-Ss ([2, 4])
  - Derivation in no need of B
    - S1. Derivation of $|a(t)|$ in the case $a(t) \in \mathcal{L}^{1,2}$ is absolutely continuous ([2])
  - Derivation in need of B
    - S2. Derivation of $a(t) \in \mathcal{L}^{1,2}$ and $b(t) \in \mathcal{L}^{0,2}$ ([4]) (extension of the results in [8, 9])
    - S3. Derivation of $a(t)$ and $b(t)$ in the case $a(t) \in \mathcal{L}^{1,2}$ is absolutely continuous ([2])
- Derivation of random functions from SFC-Os ([1, 2, 4])
  - Derivation in no need of B
    - O1. Derivation of $|a(t)|$ in the case $a(t) \in$ FVP ([2]) (extension of the result in [10])
    - O2. Derivation of $|\mathrm{Re}\,a|$, $|\mathrm{Im}\,a|$, $\mathrm{Re}\,a\,\mathrm{Im}\,a$ and $(\mathrm{sgn}\,a)a$ in the case $a(t)$ is written as $a(t) = V_t + M_t + Z_t + W_t$ with $V_t \in$ FVP, an Itô integral process M_t, a Skorokhod integral process Z_t and the Hilbert-Schmidt transform W_t of a functional in $\mathcal{L}^{1,2}$, or a more general random function ([1]) (extension of [7, Theorem 2], O1)
  - Derivation in need of B
    - O3. Derivation of $a(t)$ and $b(t)$ in the case $a(t)$ is a Skorokhod integral process and $b(t) \in \mathcal{L}^{0,2}$ ([4])
    - O4. Derivation of $a(t)$ and $b(t)$ in the case $a(t) \in$ FVP ([2])
    - O5. Derivation of $a(t)$ and $b(t)$ in the case $a(t)$ is written as $a(t) = V_t + M_t + Z_t + W_t$ with $V_t \in$ FVP, an Itô integral process M_t, a Skorokhod integral process Z_t and the Hilbert-Schmidt transform W_t of a functional in $\mathcal{L}^{1,2}$, or a more general random function ([1]) (extension of O3, O4)

References

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