

Future Research Plans

Newforms of half-integral weight and local new vectors of the metaplectic group $\text{Mp}(4)$

I will investigate newforms as a development of Ibukiyama's conjectures. There is an information called the level of modular forms. A modular form of smaller level is called a newform. In the case of degree 1, there is a study of newforms in the results of Shimura, given by Ueda-Yamana. Therefore, following this, I will study newforms in Ibukiyama's conjectures in the context of modular forms of half-integral weight and of degree 2. On the other hand, a local new vector is a vector in a representation of a group over a local field, which is a local component of an automorphic representation, corresponding to a newform. In this project, we investigate the existence and the number of local new vectors in the representations of $\text{Mp}(4)$ and newforms of half-integral weight and of degree 2.

First of all, for newform, the case of integral weight is well known, so we get results by constructing a similar isomorphism between them of half-integral weight and integral weight as Ibukiyama's conjectures. There is a prior study by Ueda-Yamana in the case of degree 1. Referring to this, I will make predictions, and prove it by applying the method of the proof ([Is2]) of Ibukiyama's conjectures. On the other hand, for local new vectors, I will extend a previous study in the case of the degree 1 by Roberts-Schmidt to the case of degree 2. Roberts-Schmidt's study uses Waldspurger's results to investigate the existence and the number of local new vectors of $\text{Mp}(2)$. I will utilize the results of Gan-Savin and Gan-Ichino, which are generalizations of the studies of Waldspurger. Finally, I will compare the results of researches of newforms and of local new vectors.

In the field of automorphic representations, there is an important conjecture called "GGP conjecture" about the automorphic representations, representations over local fields, and their L-functions. Newforms are useful for calculating the L-functions, and the information of the existence of newforms and local new vectors of small level has great value.

Beyond Ibukiyama's isomorphism

Ibukiyama's isomorphism is an isomorphism

$$S_{k-\frac{1}{2}j}^+(\Gamma_0(4), \psi) \cong S_{j+3, 2k-6}(\text{Sp}(4, \mathbb{Z})),$$

of vector spaces which preserves L-functions, for a nonnegative even integer j and an integer $k \geq 3$. Roughly speaking, the left hand side is a space of half-integral weight Siegel cusp forms, and the right hand side is a space of those of integral weight. Here, the conditions " $k \geq 3$ " and " $j \geq 0$ " are necessary, because one side becomes trivially zero. However, the reason why the integer j is required to be even is not well-known. Through the study of Ibukiyama's isomorphism, I got convinced that there exists a similar isomorphism when j is odd.

In the paper [Is2], I proved the isomorphism by specifying the automorphic representations of $\text{Mp}(4)$ and $\text{SO}(5)$ corresponding to the left and right hand side, respectively. This observation told me that there are automorphic representations of $\text{Mp}(4)$ which should appear in the case j is odd. I will study these representations precisely to construct an isomorphism similar to Ibukiyama's.