Research Plan

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In my future research, I'm planning to focus on the following two issues: First, I will study the regularity of the dual process of the jump-diffusion processes, which is currently sealed in my study. The purpose of this study is to investigate the smoothness with respect to the starting point of the density function of the jump-diffusion process.
The construction and regularity of dual processes for jump diffusion processes on the Euclidean space are described in Kuniata (2019). We aim to extend this method to jump-diffusion processes on manifolds. If we restrict the Riemannian manifold to an Hadamard manifold, it becomes diffeomorphic to the Euclidean space and we can use the method described in Kuniata (2019), so we will first study the dual process of the jump diffusion process on Hadamard manifolds.

The second is to study the long-time behavior of jump-diffusion processes on more general manifolds. In our previous study, we confirmed that the long-time behavior can be captured by evaluating the radial part of the jump-diffusion process on Hadamard manifolds whose sectional curvature is pinched by two negative constants. However, the assumption that the sectional curvature of the manifold is supported from below by a negative constant is a rather stringent condition. To solve this issue, I'm planning to elucidate how the jump diffusion process is affected by the curvature of the manifold more clearly. The following three specific objectives are set for this study.

1. obtain an evaluation for the radial part of the jump-diffusion process on a positive curvature manifold.
2. find the conditions of curvature of the manifold for the jump diffusion process to be recurrent.
3. obtain the conditions for a jump diffusion process to have the Feller property (and the strong Feller property).

To achieve these goals, we will extend the method of evaluating the radial part of the jump-diffusion process from previous studies to seek an evaluation that reflects more detailed information on the curvature of the manifold, and find the evaluation of the time at which the jump diffusion process escapes from a geodesic ball.

Once these studies are completed, we will be able to study matters that have been studied using Dirichlet forms from the viewpoint of stochastic differential equations.